1. The matrix $A=\begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$ has characteristic polynomial $-(\lambda-3)^2(\lambda-8)$.

   a. Find a basis for the eigenspace corresponding to the eigenvalue $\lambda=3$.

   b. Is A diagonalizable? Explain briefly.

2. Given the system

$$\begin{aligned} x_1+3x_2&=2 \\ 3x_1+hx_2&=k \end{aligned}$$

   for what values of h and k will the system have a unique solution?

3. Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2 - 2x_3, \quad -x_1 + 2x_2 + 3x_3)$.
   a. What properties would we have to check to show that T a linear transformation? (If you finish the rest of the worksheet, come back and show them.)

   b. Find $\ker T$, a basis for $\ker T$, and the dimension of $\ker T$.

   c. Is T onto? (Justify.)

4. Let A be a 42×35 matrix, and let $T:\mathbb{R}^{35} \to \mathbb{R}^{42}$ be the transformation $T(\mathbf{x}) = A\mathbf{x}$.
   a. Suppose A has 30 pivot columns. Find $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$.

   b. If $\operatorname{Col} A$ is a subspace of $\mathbb{R}^j$ and $\operatorname{Nul} A$ is a subspace of $\mathbb{R}^k$, find j and k.

5. In $\mathbb{R}^3$, let $W = \text{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$. Let $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find the closest point to $\mathbf{y}$ in W and the distance between $\mathbf{y}$ and W. (No need to simplify the ugly fractions.) Don't forget the first step.

6. Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ be bases of $\mathbb{R}^2$. Find the change of basis matrix from $\mathcal{B}$ to $\mathcal{C}$ and the change of basis matrix from $\mathcal{C}$ to $\mathcal{B}$. ***Hint***: Once you find the first matrix, the second is the inverse.

7. Let $T:\mathbb{P}_2 \to \mathbb{P}_1$ be defined using the derivative, so $T(f) = f'$.
   a. Find the matrix for T relative to the bases $\{t^2+2, t+3, t+1\}$ and $\{t,1\}$.

   b. Find the kernel of T. (nothing to do with the basis in part (a); this is a new question.)

   c. Find the range of T.

8. Consider the matrix $A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$. Find the eigenvalues, and for one of them (your choice) find a corresponding **eigenvector**.

9. Decide whether or not each subset is a subspace. (You should be prepared to justify whether it satisfies ***each*** of the three properties, and give an example if it does not.)
   a. Let H be the set of matrices in $M_{2\times 2}$ satisfying $\det(A) = 0$.
   b. Let H be the set of polynomials in $\mathbb{P}_3$ satisfying $p(5) = p(2)$.