These problems will help you review for exam 2. This is not a comprehensive review; it is just meant to help you get started.

1. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 6 \\ 3 \\ 0 \end{bmatrix}$.

   a. Let $\mathbf{y} = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 5 \end{bmatrix}$. Write $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\hat{\mathbf{y}}$ is the orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$.

   b. Find all vectors in $\mathbb{R}^4$ which are orthogonal to $\mathbf{v}$.

2. In each case, decide if the matrix M is diagonalizable, is not diagonalizable, or if there is not enough information to decide.

   a. M is a 3×3 matrix with eigenvalues 0, 4, and 8.

   b. M is a 4×4 matrix with eigenvalues 1, 4 and 8.

   c. M is a 5×5 matrix with three eigenvalues, two of which have 1-dimensional eigenspaces and one with a 2-dimensional eigenspace.

3. The matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}$ has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 1$ and $\lambda_3 = 4$. Diagonalize A.

   Hint: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}$ are both eigenvectors.

4. Let $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 1 & 2 & 3 & -1 \\ -2 & -4 & -6 & 2 \end{bmatrix}$.

   a. Find a basis for $\text{Col}\,A$.

   b. Find a basis for $\text{Nul}\,A$.

   Let $\mathbf{x} = \begin{bmatrix} -4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Verify $\mathbf{x}$ is in $\text{Nul}\,A$, and then find the coordinate vector for $\mathbf{x}$ in terms of your basis for $\text{Nul}\,A$.

   c. Is your basis for $\text{Nul}\,A$ orthogonal? Is it orthonormal? Explain briefly.

5. Let $W = \text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ where $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$. Use the Gramm-Schmidt process to produce a new basis for W. What property does this new basis have that the original does not?