

These problems should help you review for our first exam. They are not meant to be comprehensive. You should refer to your **class notes, other worksheets, online homework and written homework** to help prepare for the exam.

1. Suppose a system of equations has the augmented matrix $\left[\begin{array}{cc|c} h & 1 & 3 \\ 1 & 2 & k \end{array} \right]$.

Find **all** values of h and k for which the system has

(a) infinitely many solutions, (b) no solutions, or (c) a unique solution. Explain briefly.

2. Let $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be a linear transformation defined by

$T(x_1, x_2, x_3, x_4) = (2x_1, -x_3 + x_4, x_2 + x_3 - 2x_4)$. Find the standard matrix A associated to T .

3. Find the standard matrix A for the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that **first** reflects points through the line $y = x$ and **then** reflects through the vertical y -axis.

4. If A is a 5×5 matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^5 , is it possible that for some \mathbf{b} , the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution? Why or why not?

5. Suppose we have a transformation $T: \mathbb{R}^5 \rightarrow \mathbb{R}^3$ given by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}. \text{ For each of these answer Yes or No; then explain briefly.}$$

- a. Is the vector $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$ in the image of T ?
- b. Is the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the image of T ?

- c. Is T one-to-one? (explain briefly)
- d. Is T onto? (explain briefly)
6. Can a square matrix with two identical columns be invertible? Why or why not?
7. Can a square matrix with two identical rows be invertible? Why or why not?