

The matrix

$$A = \begin{bmatrix} 3 & -6 & 9 & 0 & 3 \\ 2 & -4 & 7 & 2 & 0 \\ 3 & -6 & 6 & -6 & 0 \end{bmatrix} \text{ is row-equivalent to the matrix } \begin{bmatrix} 1 & -2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

1. Write a basis for  $\text{Col } A$ .

2. What is the rank of  $A$ ?

3. Are  $\mathbf{p}_1 = \begin{bmatrix} 8 \\ 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$ ,  $\mathbf{p}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{p}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  in  $\text{Nul } A$ ? (Justify.)

4. Find a basis for  $\text{Nul } A$ .

5. Find the dimension of  $\text{Nul } A$ .

6. Suppose  $A$  is an  $8 \times 20$  matrix with seven pivot columns. Find the dimensions of  $\text{Col } A$  and  $\text{Nul } A$ .

7. Let  $H = \left\{ \begin{bmatrix} p+r \\ 2p \\ 2r \\ 6p+2r \end{bmatrix} \mid p, r \text{ are real numbers} \right\}$ . Show that  $H$  is a subspace of  $\mathbb{R}^4$ .

Hint: Write  $H$  as a span of vectors.

8. Let  $H = \left\{ \begin{bmatrix} p+r \\ p+2 \\ r+1 \\ 6p+2r \end{bmatrix} \mid p, r \text{ are real numbers} \right\}$ . Show that  $H$  is not a subspace of  $\mathbb{R}^4$  by

clearly showing which property or properties it violates.