The Invertible Matrix Theorem

Let A be a square $n \times n$ matrix. The following statements are equivalent. That is, for a given A, the statements are either all true or all false.

a. A is an invertible matrix.
b. A is row equivalent to the $n \times n$ identity matrix.
c. A has n pivot positions.
d. The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
e. The columns of A form a linearly independent set.
f. The linear transformation $\mathbf{x} \to A\mathbf{x}$ is one-to-one.
g. The equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
h. The columns of A span $\mathbb{R}^n$.
i. The linear transformation $\mathbf{x} \to A\mathbf{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.
j. There is an $n \times n$ matrix C such that $CA = I$.
k. There is an $n \times n$ matrix D such that $AD = I$.
l. A^T is an invertible matrix.

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m. The columns of A form a basis of $\mathbb{R}^n$.
n. $\operatorname{Col} A = \mathbb{R}^n$
o. $\dim \operatorname{Col} A = n$
p. $\operatorname{rank} A = n$
q. $\operatorname{Nul} A = \{\mathbf{0}\}$
r. $\dim \operatorname{Nul} A = 0$
s.
t.