

Let $T(\mathbf{x}) = A\mathbf{x}$ be a linear transformation with $A = \begin{bmatrix} 1 & -3 & 2 \\ 3 & -8 & 8 \\ 0 & 1 & 2 \\ 1 & 0 & 8 \end{bmatrix}$.

1. If $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$, find n and m .

2. Let $\mathbf{b} = \begin{bmatrix} 1 \\ 6 \\ 3 \\ 10 \end{bmatrix}$. Find all possible \mathbf{x} with $T(\mathbf{x}) = \mathbf{b}$.

Hint: write the augmented matrix with the columns of A and \mathbf{b} , and row-reduce.

3. Do the columns of A span all of \mathbb{R}^4 ? (i.e., can we find \mathbf{x} with $T(\mathbf{x}) = \mathbf{b}$ for all choices of \mathbf{b} in \mathbb{R}^4 ?) Explain. *Hint:* the row-reduction you did in question 2 will be helpful.
4. Are the columns of A linearly independent? If not, find a dependence relation. *Hint:* the row-reduction you did in question 2 will be helpful.