Let $W = \text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ where $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

a. Check that this spanning set is an orthogonal basis for W.

b. Let $\mathbf{w} = \begin{bmatrix} 8 \\ -6 \\ -2 \\ 16 \end{bmatrix}$. Compute $[\mathbf{w}]_{\mathcal{B}}$ (using the formula that works only for an orthogonal basis).

c. An **orthonormal basis** is a basis that is orthogonal, in which every vector has length 1. Find an orthonormal basis for W. (Replace each $\mathbf{u}_i$ with a unit vector pointing in the same direction.)

We will still let $W = \text{span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ where $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

d. Let $\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}$. Compute $\text{proj}_{\mathbf{u}_1} \mathbf{y}$, $\text{proj}_{\mathbf{u}_2} \mathbf{y}$ and $\text{proj}_{\mathbf{u}_3} \mathbf{y}$.

e. Find the point in W that is closest to $\mathbf{y}$, and find the distance between W and $\mathbf{y}$.