Please write up complete, clear solutions. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly.

This assignment comes from material in sections 1.1, 1.2 and 1.3. The textbook will be a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or possibly in the Mathskeller (depending on who is tutoring at the time).

1. Suppose we know the system

$$\begin{aligned} 3x_1 + 5x_2 &= h \\ cx_1 + dx_2 &= k \end{aligned}$$

   is consistent for all possible values of h and k. What can you say about c and d?

2. Find three different augmented matrices which represent linear systems with solutions $x_1 = 6$, $x_2 = 2$ and $x_3 = 4$.

3. In the following matrices, ■ represents a nonzero entry, and * represents an entry that may or may not be zero. For each of these **augmented** matrices determine if the associated linear system is inconsistent, consistent with a unique solution, or consistent with infinitely many solutions. (*Hint*: it may help to start by drawing the vertical bar separating coefficients from constants.)

   a. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \end{bmatrix}$

   b. $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$

   c. $\begin{bmatrix} \blacksquare & 0 \\ 0 & 0 \end{bmatrix}$

   d. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

   e. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

4. A system with fewer equations than unknowns is called an ***underdetermined*** system. A system with more equations than unknowns is called an ***overdetermined*** system.
   a. Give an example of an inconsistent underdetermined linear system.

   b. Give an example of a consistent overdetermined linear system.

5. **True or False**. Decide if each statement is true (always true) or false (can ever be false). If true explain clearly. If false, explain or give a counterexample where appropriate.
   a. Points in the plane corresponding to the vectors $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ line on a line through the origin.

   b. An example of a linear combination of the vectors $\mathbf{u}$ and $\mathbf{v}$ is $\frac{2}{3}\mathbf{v}$.

   c. The set $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.