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MA 322: Written Homework 1

+6 1. Suppose we know the system

$3x_1 + 5x_2 = h$
$cx_1 + dx_2 = k$

is consistent for all possible values of h and k. What can you say about c and d?

$$\left[\begin{array}{cc|c} 3 & 5 & h \\ c & d & k \end{array}\right]$$

$$\left[\begin{array}{cc|c} 3 & 5 & h \\ 0 & d - 5/3c & k - \frac{h}{3}c \end{array}\right] \quad R_2 \leftarrow R_2 - \frac{c}{3}R_1$$

The system would be consistent when $d - 5/3c \neq 0$ since we can always find an h and k so $k - h/3c \neq 0$. great!

+6 2. Find three different augmented matrices which represent linear systems with solutions $x_1 = 6$, $x_2 = 2$, and $x_3 = 4$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} x_1 = 6 \\ x_2 = 2 \\ x_3 = 4 \end{array}$$ ✓

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 12 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 8 \end{array}\right]$$ ✓ If you divide each row by 2 then the solution is $x_1 = 6$, $x_2 = 2$, and $x_3 = 4$.

$$\left[\begin{array}{ccc|c} 5 & 0 & 0 & 30 \\ 0 & 5 & 0 & 10 \\ 0 & 0 & 5 & 20 \end{array}\right]$$ ✓ If you divide each row by 5 then the solution is $x_1 = 6$, $x_2 = 2$, and $x_3 = 4$.

3. In the following matrices, $\blacksquare$ represent a nonzero entry, and $*$ represents an entry that may or may not be zero. For each of these augmented matrices determine if the associated linear system is inconsistent, consistent with a unique solution, or consistent with infinitely many solutions.

a) $$\begin{array}{c} \begin{array}{cc|c} x_1 & x_2 & b \end{array} \\ \left[\begin{array}{cc|c} \blacksquare & * & * \\ 0 & \blacksquare & * \end{array}\right] \end{array}$$

Consistent with a unique solution because x_1 and x_2 are both basic variables

b) $$\begin{array}{c} \begin{array}{c|c} x_1 & b \end{array} \\ \left[\begin{array}{c|c} \blacksquare & * \\ 0 & \blacksquare \end{array}\right] \end{array}$$

Inconsistent with no solutions because Row 2 is $[0 \mid b]$ $b \neq 0$

c) $$\begin{array}{c} \begin{array}{c|c} x_1 & b \end{array} \\ \left[\begin{array}{c|c} \blacksquare & 0 \\ 0 & 0 \end{array}\right] \end{array}$$

Consistent with a unique solution because x_1 is a basic variable.

d) $$\begin{array}{c} \begin{array}{ccc|c} x_1 & x_2 & x_3 & b \end{array} \\ \left[\begin{array}{ccc|c} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{array}\right] \end{array}$$

Consistent with infinitely many solutions because x_1 is a free variable. x_2 and x_3 are basic variables.

e) $$\begin{array}{c} \begin{array}{ccc|c} x_1 & x_2 & x_3 & b \end{array} \\ \left[\begin{array}{ccc|c} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{array}\right] \end{array}$$

Inconsistent with no solutions because Row 3 is $[0\,0\,0 \mid b]$ and $b \neq 0$.

great explanations!

4. A system with fewer equations than unknowns is called an underdetermined system. A system with more equations than unknowns is called an overdetermined system.

a) Give an example of an inconsistent underdetermined linear system.

$x_1 + x_2 + x_3 = 1$ ✓
$x_1 + x_2 + x_3 = 3$

This is an example of an inconsistent underdetermined linear system because there are 2 equations and 3 unknowns (fewer equations than unknowns and when I solved the matrix (shown below) I got [0 0 0 | b] where b≠0 in Row 2.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right] R_2 \leftarrow R_2 - R_1$$

fantastic explanations!

b) Give an example of a consistent overdetermined linear system.

$2x_1 + x_2 = 2$
$-4x_1 - 4x_2 = -24$
$x_1 + x_2 = 6$

This is an example of a consistent overdetermined matrix because there are more equations (there are 3) than variables (there are 2) and there's a solution (as shown below).

$$\left[\begin{array}{cc|c} 2 & 1 & 2 \\ -4 & -4 & -24 \\ 1 & 1 & 6 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ -4 & -4 & -24 \\ 1 & 1 & 6 \end{array}\right] R_1 \leftarrow R_1 - R_3$$

$$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ -4 & -4 & -24 \\ 4 & 4 & 24 \end{array}\right] 4R_3$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & -4 \\ -4 & -4 & -24 \\ 0 & 0 & 0 \end{array}\right] R_3 \leftarrow R_3 + R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ 1 & 1 & 6 \\ 0 & 0 & 0 \end{array}\right] -\tfrac{1}{4}R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 10 \\ 0 & 0 & 0 \end{array}\right] R_2 \leftarrow R_2 - R_1$$

$$\boxed{x_1 = -4, \quad x_2 = 10}$$

+6 5. True or False. Decide if each statement is true (always true) or false (can ever be false). If true explain clearly. If false, explain or give a counterexample where appropriate.

a) Points in the plane corresponding to the vectors $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ line on a line through the origin.

False because we have a setup like $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and we'd need $\vec{v}$ to be a multiple of $\vec{u}$, which it's not.

$c\vec{v} = \vec{u}$

$\left.\begin{array}{l} c(-2) = 3 \\ c(-3) = 2 \end{array}\right\}$ there is not a c to satisfy both of these equations.

b) An example of a linear combination of the vectors $\vec{u}$ and $\vec{v}$ is $\frac{2}{3}\vec{v}$.

True, if both $\vec{u}$ and $\vec{v}$ are in $\mathbb{R}^n$. For example, $0\vec{u} + \frac{2}{3}\vec{v} = \frac{2}{3}\vec{v}$, showing if you multiply $\vec{u}$ by a constant, zero, and $\vec{v}$ by a constant $\frac{2}{3}\vec{v}$, then when you add them you get $\frac{2}{3}\vec{v}$.

c) The set Span$\{\vec{u}, \vec{v}\}$ is always visualized as a plane through the origin.

False. Span$\{\vec{u}, \vec{v}\}$ is a plane through the origin unless $\vec{u}$ is a multiple of $\vec{v}$ or $\vec{u}, \vec{v} = \vec{0}$. ($\vec{0}$ = zero vector).