Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly. For the constructions, you should justify that your construction meets the required criteria.

This assignment covers material in sections 2.8 and 2.9. The textbook is a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or possibly in the Mathskeller (depending on who is tutoring at the time).

1. Let $H$ be the set in $\mathbb{R}^{3}$ that contains only the zero vector.
a. Show directly from the properties that $H$ is a subspace of $\mathbb{R}^{3}$.
b. Can you find a basis for $H$ ? Explain.
2. Construct a $3 \times 4$ matrix $A$ and a vector $\mathbf{b}$ with $\mathbf{b}$ not in $\operatorname{Col} A$.
3. Construct a $4 \times 5$ matrix $A$ with $\operatorname{dim} \operatorname{Nul} A=3$ and $\operatorname{dim} \operatorname{Col} A=2$.
4. Let $A$ be a $6 \times 4$ matrix with linearly independent columns. What can we say about $\operatorname{Nul} A$ ?
5. What is the rank of a $7 \times 9$ matrix whose null space is 4 -dimensional?
6. Let $\mathcal{B}$ be a basis for $\mathbb{R}^{2}$ given by $\mathcal{B}=\left\{\left[\begin{array}{l}2 \\ 3\end{array}\right],\left[\begin{array}{r}-3 \\ 1\end{array}\right]\right\}$.
a. Find the vector $\mathbf{x}$ with $[\mathbf{x}]_{\mathscr{B}}=\left[\begin{array}{r}2 \\ -1\end{array}\right]$.
b. Find the coordinate vector $[\mathbf{x}]_{\mathscr{B}}$ for the vector $\mathbf{x}=\left[\begin{array}{l}-8 \\ -1\end{array}\right]$.
