MA 322

$\frac{24}{24}$

Written HW 4 3/2/17

1. H is the set in $\mathbb{R}^3$ that contains only the zero vector

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$$H = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

a. For H to be a subspace:

① Is $\vec{0}$ in H?

Yes, $\vec{0}$ is very clearly in H

② Closed under addition?

$\vec{u} = [0\ 0\ 0]$ $\vec{v} = [0\ 0\ 0]$

$\vec{u} + \vec{v} = [0\ 0\ 0]$ ✓

③ Closed under scalar multiplication?

$\vec{u} = [0\ 0\ 0]$, c any real #

$c\vec{u} = [0\ 0\ 0]$ ✓

H is a subspace of $\mathbb{R}^3$

b. A basis is a linearly independent set in H that spans H. Since the only vector in H is $\vec{0}$, and $\vec{0}$ isn't linearly independent, H does not have a basis.

2. A is 3×4, make $\vec{b}$ <u>not</u> in Col A

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Col A is the span of the columns of A,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

so if b isn't the span of A (not consistent) it won't be in Col A

3. A is 4×5, $\dim \operatorname{Nul} A = 3$, $\dim \operatorname{Col} A = 2$

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This means A needs 3 free variables and 2 basic variables.

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. A is 6×4 with linearly independent columns. ~~Nul A is the set of all solutions to $A\vec{x} = \vec{0}$.~~ Since the columns (c_i) of A are linearly independent, the vector equation $x_1\vec{c}_1 + x_2\vec{c}_2 + x_3\vec{c}_3 + x_4\vec{c}_4 = 0$ has only the trivial solution. Therefore $\vec{x} = \vec{0}$. And because Nul A is the set of all solutions to $A\vec{x} = \vec{0}$,

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$$\operatorname{Nul} A = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

5. By the Rank-Nullity Theorem,

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$$n = \operatorname{rank} A + \dim \operatorname{Nul} A$$

A is 7×9
$\hookrightarrow n = 9$

$\dim \operatorname{Nul} A = 4$

$$9 = \operatorname{rank} A + 4$$

$$\boxed{\operatorname{rank} A = 5}$$

6. $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$ and is a basis for $\mathbb{R}^2$

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a. Find $\vec{x}$ with $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$$\vec{x} = 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 1\begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+3 \\ 6-1 \end{bmatrix}$$

$$\boxed{\vec{x} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}}$$

b. $\vec{x} = \begin{bmatrix} -8 \\ -1 \end{bmatrix}$ Find $[\vec{x}]_{\mathcal{B}}$

$$\left[\begin{array}{cc|c} 2 & -3 & -8 \\ 3 & 1 & -1 \end{array}\right] \xrightarrow{R_1 = \frac{1}{2} r_1} \left[\begin{array}{cc|c} 1 & -3/2 & -4 \\ 3 & 1 & -1 \end{array}\right] \xrightarrow{R_2 = r_2 - 3r_1} \left[\begin{array}{cc|c} 1 & -3/2 & -4 \\ 0 & 11/2 & 11 \end{array}\right]$$

$$\xrightarrow{R_2 = 2/11\, r_2} \left[\begin{array}{cc|c} 1 & -3/2 & -4 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R_1 = r_1 + \frac{3}{2} r_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$$

$$\vec{x} = -1\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

so $\boxed{[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}}$