Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly. For the constructions, you should justify that your construction meets the required criteria.

This assignment covers material in sections 3.1, 3.2, and 5.1.

1. It usually not the case that $\det(A+B) = \det A + \det B$. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

   Show that $\det(A+B) = \det A + \det B$ if and only if $a+d=0$.

2. Suppose A and P are $n \times n$ matrices with P invertible. Show that $\det(PAP^{-1}) = \det A$.

3. Suppose A is an $n \times n$ matrix with $A^T A = I_n$. What are the possible values of $\det A$?

4. Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = 4$ for the matrix

$$A = \begin{bmatrix} 5 & 0 & -1 & 0 \\ 1 & 3 & 0 & 0 \\ 2 & -1 & 3 & 0 \\ 4 & -2 & -2 & 4 \end{bmatrix}$$

5. Construct a 2×2 matrix with one distinct eigenvalue.

6. Just like there is a special formula to compute a determinant of a 2×2 matrix, there is also a specific technique to find the determinant of a 3×3 matrix.
   a. In Section 3.1, read about this technique, which is described before exercises 15-18. Work problems 15 and 17 with this technique and check your answers. (You do not need to turn these in. Answers will vary depending on which edition you have.)
   b. Use this determinant technique when finding the characteristic equation for the matrix

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}.$$