Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly. **For the construction, you should justify that your construction meets the required criteria.**

This assignment covers material in sections 5.3, 5.4, and 5.5.

1. Suppose A is a 5×5 matrix with exactly three eigenvalues, and we know one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

2. Now suppose A is a 6×6 matrix with exactly three eigenvalues, and one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

3. Construct an example of a matrix that is diagonalizable, is not diagonal, and is not invertible.

4. Let $A=\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$, and $\mathcal{B}=\{\mathbf{b}_1,\mathbf{b}_2\}$ where $\mathbf{b}_1=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{b}_2=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$. Define $T:\mathbb{R}^2\to\mathbb{R}^2$ by $T(\mathbf{x})=A\mathbf{x}$. Find $[T]_{\mathcal{B}}$.

5. Let $A=\begin{bmatrix} \sqrt{3} & 3 \\ -3 & \sqrt{3} \end{bmatrix}$. The transformation $\mathbf{x}\mapsto A\mathbf{x}$ is a composition of a rotation and a scaling.
    a. Give the angle of rotation and the scale factor.
    b. Find the eigenvalues and a basis for each eigenspace.