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Written Assignment 6 for MA 322 - Matrix Algebra (Spring 2017)

March 30, 2017

Problem 1

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Suppose A is a 5×5 matrix with exactly three eigenvalues, and we know one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

If A is a 5×5 matrix with exactly three eigenvalues, then we know that there are exactly three eigenspaces, each of which is at least one-dimensional. We also know that the sum of the dimensions of all eigenspaces is less than or equal to the number of columns in the original matrix. If one eigenspace is known to be three-dimensional, then in order for the dimensions of all three eigenspaces to add up to 5 we know that each of the other two eigenspaces must be exactly one-dimensional. Since we know that each eigenspace is at least one-dimensional without having to verify it using the exact eigenvectors, we $\boxed{\text{can}}$ immediately determine that A is diagonalizable.

Problem 2

Now suppose A is a 6×6 matrix with exactly three eigenvalues, and one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

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This time, in order for the dimensions of all three eigenspaces to add up to 6, one of the two remaining eigenspaces must be one-dimensional, and the other two-dimensional. Since the existence of a two-dimensional eigenspace cannot be verified without checking the eigenvectors, we $\boxed{\text{cannot}}$ determine whether or not A is diagonalizable without more information.

Problem 3

Construct an example of a matrix that is diagonalizable, is not diagonal, and is not invertible.

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In order for a matrix to be diagonalizable, it must have exactly as many linearly independent eigenvectors as columns. In order for the matrix to also not be invertible, it must have linearly dependent rows, which always happens whenever one of the eigenvalues is 0. Therefore, one example of a diagonalizable (yet not diagonal) matrix with 0 as one of the eigenvalues is a 2×2 matrix with eigenvalues 0 and 1, or characteristic equation $\lambda^2 - \lambda = 0$. Creating a matrix with this equation is relatively easy - here is one such example:

$$\boxed{\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}} \text{(characteristic equation} = \left(\lambda^2 - \lambda + \frac{1}{4}\right) - \left(\frac{1}{4}\right) = 0 \Leftrightarrow \lambda^2 - \lambda = 0)$$

Problem 4

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Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$, and $\mathscr{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ where $\boldsymbol{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{b}_2 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$. Find $[T]_{\mathscr{B}}$.

Step 1: Figure out the linear transformation of each basis vector:

$$T(\mathbf{b}_1) = \begin{bmatrix} 1 \cdot 1 + 1 \cdot 1 \\ -1 \cdot 1 + 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$T(\mathbf{b}_2) = \begin{bmatrix} 1 \cdot 5 + 1 \cdot 4 \\ -1 \cdot 5 + 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

Step 2: Write these vectors with respect to $\mathscr{B}$, and make them the columns of $[T]_{\mathscr{B}}$:

$$[T(\mathbf{b}_1)]_{\mathscr{B}} = \text{solution to } \left[\begin{array}{cc|c} 1 & 5 & 2 \\ 1 & 4 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 5 & 2 \\ 0 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 5 & 2 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$[T(\mathbf{b}_2)]_{\mathcal{B}} = \text{solution to } \left[\begin{array}{cc|c} 1 & 5 & 9 \\ 1 & 4 & 7 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 5 & 9 \\ 0 & -1 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 5 & 9 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right] = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \boxed{[T]_{\mathcal{B}} = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}}$$

Problem 5

Let $A = \begin{bmatrix} \sqrt{3} & 3 \\ -3 & \sqrt{3} \end{bmatrix}$. *The transformation* $\boldsymbol{x} \mapsto A\boldsymbol{x}$ *is a composition of a rotation and a scaling.*

a. *Give the angle of rotation and the scale factor.*

Scale factor (s) = norm of every column vector in A:

$$s = ||\mathbf{a}_1|| = \sqrt{(\sqrt{3})^2 + (-3)^2} = \sqrt{12} = \boxed{2\sqrt{3}}$$

Dividing A by s makes the rotation easier to see:

$$A = 2\sqrt{3}\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}.$$

Since the rotation matrix is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $\cos\theta > 0$ and $\sin\theta < 0$, putting θ in Quadrant IV. Applying the appropriate reference angle, we get

$$A = 2\sqrt{3}\begin{bmatrix} \cos -\frac{\pi}{3} & -\sin -\frac{\pi}{3} \\ \sin -\frac{\pi}{3} & \cos -\frac{\pi}{3} \end{bmatrix}$$

which means that $\boxed{\theta = -\frac{\pi}{3}}$

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b. *Find the eigenvalues and a basis for the eigenspace.*

Because this is a rotation, the eigenvalues are going to be imaginary:

$$A - \lambda I = 0 \Leftrightarrow \begin{vmatrix} \sqrt{3} - \lambda & 3 \\ -3 & \sqrt{3} - \lambda \end{vmatrix} = 0 \Leftrightarrow \left(\sqrt{3} - \lambda\right)^2 + (3)^2 = 0$$

$$\Leftrightarrow 3 - 2\sqrt{3}\lambda + \lambda^2 + 9 = 0 \Leftrightarrow \lambda^2 - 2\sqrt{3}\lambda + 12 = 0$$

$$\Rightarrow \lambda = \frac{2\sqrt{3} \pm \sqrt{12 - 48}}{2} = \sqrt{3} \pm 3i$$

The eigenvectors can be computed in a very straightforward manner (NOTE: Only the computation for $\lambda = \sqrt{3} + 3i$ is shown - since A contains only real numbers, the other eigenvector must be the conjugate of this one.)

$$A - \lambda I = \begin{bmatrix} \sqrt{3} - \sqrt{3} - 3i & 3 \\ -3 & \sqrt{3} - \sqrt{3} - 3i \end{bmatrix} = \begin{bmatrix} -3i & 3 \\ -3 & -3i \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3i & 3 \\ -3i & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -3i & 3 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow x_1 = -ix_2 \Rightarrow \text{Nul}\, A = \text{Span}\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$$

Thus, the eigenvector for $\lambda_1 = \sqrt{3} + 3i$ is $\begin{bmatrix} -i \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$, and the eigenvector for $\lambda_1 = \sqrt{3} - 3i$ is $\begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$. Any matrix involving only rotation and scaling has these two eigenvectors. Therefore, the basis for the eigenspace is $\boxed{\left\{\begin{bmatrix} 1 \\ i \end{bmatrix}, \begin{bmatrix} 1 \\ -i \end{bmatrix}\right\}}$.

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Written HW 6 3/30/17

1. A is 5 x 5, 3 eigenvalues, one eigenspace is three-dimensional; can we determine if A is diagonalizable?

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A will only be diagonalizable if it has 5 linearly independent eigenvectors. A only has three eigenvalues, but as long as the multiplicities of these eigenvalues adds to five, A will be diagonalizable. Since one eigenvalue has a three-dimensional eigenspace, and we can assume the other two have one-dimensional eigenspaces, leading to five linearly independent eigenvectors, making A diagonalizable

2. A is 6x6, 3 eigenvalues, one eigenspace is three-dimensional; can we determine if A is diagonalizable?

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Using the same reasoning as in #1, if the other two eigenvalues only have one eigenvector a piece, there won't be enough eigenvectors to make A diagonalizable. So we can not determine if it is or not.

3. $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} = A$

6/6 $\begin{bmatrix} 2-\lambda & 6 \\ 1 & 3-\lambda \end{bmatrix}$ $\det = 6 - 5\lambda + \lambda^2 - 6 = 0$

$\lambda(\lambda - 5) = 0$

$\lambda = \{0, 5\}$

$\lambda = 0$ $\left[\begin{array}{cc|c} 2 & 6 & 0 \\ 1 & 3 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 1 & 3 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\tilde{x} = x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ ✓

$\lambda = 5$ $\left[\begin{array}{cc|c} -3 & 6 & 0 \\ 1 & -2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\tilde{x} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ✓

The two eigenvalues have 1-D eigenspaces so A is diagonalizable

$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{6-6} \begin{bmatrix} \ldots \end{bmatrix} = \frac{1}{0} \begin{bmatrix} \ldots \end{bmatrix}$

A is not invertible b/c det A = 0

4. $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ $\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\vec{b}_2 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(\vec{x}) = A\vec{x}$, Find $[T]_{\mathcal{B}}$

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Using the formula: $D = C^{-1}AC$, where C represents a change of base matrix for $\mathcal{B}$, and D represents $[T(\vec{x})]_{\mathcal{B}} = D[\vec{x}]_{\mathcal{B}}$

$$C = \begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix} \quad C^{-1} = \frac{1}{4-5}\begin{bmatrix} 4 & -5 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}$$

$$D = \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4-5 & -4+15 \\ 1+1 & 1-3 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 11 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -9+11 & -45+44 \\ 2-2 & 10-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$

$$[T(\vec{x})]_{\mathcal{B}} = \underbrace{[T]_{\mathcal{B}}}_{= D}[\vec{x}]_{\mathcal{B}}$$

so $[T]_{\mathcal{B}} = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$

5. $A = \begin{bmatrix} \sqrt{3} & 3 \\ -3 & \sqrt{3} \end{bmatrix}$ $\vec{x} \mapsto A\vec{x}$ is a composition of a rotation and a scaling

a.) Give the angle of rotation and the scale factor

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $r = \sqrt{a^2 + b^2}$

$r = \sqrt{3+9}$

$\boxed{r = \sqrt{12}}$ $\rightarrow$ $\cos\theta = a/r$

$\theta = \arccos \sqrt{3}/\sqrt{12}$

$C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ $\theta = \arccos \frac{1}{2}$

$\boxed{\theta = -\pi/3}$

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$\boxed{C = \begin{bmatrix} \sqrt{3} & -3 \\ 3 & \sqrt{3} \end{bmatrix}}$

b) Find the eigenvalues and a basis for each eigenspace

$\begin{bmatrix} \sqrt{3}-\lambda & 3 \\ -3 & \sqrt{3}-\lambda \end{bmatrix}$ $\det = (\sqrt{3}-\lambda)^2 + 9 = 0$

$(3 - 2\sqrt{3}\lambda + \lambda^2) + 9 = 0$

$\lambda^2 - 2\sqrt{3}\lambda + 12 = 0$

$\lambda = \frac{2\sqrt{3} \pm \sqrt{12-48}}{2} = \frac{2\sqrt{3} \pm 6i}{2} = \boxed{\sqrt{3} \pm 3i}$

$\underline{\lambda = \sqrt{3} + 3i}$: $\left[\begin{array}{cc|c} -3i & 3 & 0 \\ -3 & -3i & 0 \end{array}\right]$ $R_1 = R_2$

$-3ix_1 + 3x_2 = 0 \rightarrow x_1 = x_2/i \rightarrow \boxed{\text{Span}\left\{\begin{bmatrix} 1 \\ i \end{bmatrix}\right\}}$

$\underline{\lambda = \sqrt{3} - 3i}$: $\left[\begin{array}{cc|c} 3i & 3 & 0 \\ -3 & 3i & 0 \end{array}\right]$ $R_1 = R_2$

$3ix_1 + 3x_2 = 0 \rightarrow x_1 = -x_2/i \rightarrow \boxed{\text{Span}\left\{\begin{bmatrix} +1 \\ -i \end{bmatrix}\right\}}$

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Written Homework 6 009

① Suppose A is a 5x5 matrix with exactly three eigenvalues, and we know one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

A three-dimensional eigenspace means that one of the eigenvalues produced 3 linearly independent eigenvectors. By definition, we know that the other two eigenvalues each produce at least 1 eigenvector in their eigenspace, otherwise they wouldn't be eigenvalues. This brings the total eigenvectors up to 5. For a 5x5 matrix, we must have 5 linearly independent eigenvectors. As long as those 5 are linearly independent, A is diagonalizable.

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② Now suppose A is a 6x6 matrix with exactly three eigenvalues, and one of the eigenspaces is three-dimensional (we aren't told about the others). Can we determine whether or not A is diagonalizable? Explain why or why not.

A three-dimensional eigenspace means 3 linearly independent eigenvectors. By definition, we know that the other two eigenvalues produce at least 1 eigenvector, otherwise they would not be eigenvalues. This brings the minimum up to 5 eigenvectors. For a 6x6 matrix, we need 6 eigenvectors to be diagonalizable; therefore, one of those other eigenvalues needs to have an eigenspace that is two-dimensional. If it is, A is diagonalizable. If both of the other eigenvalues are one-dimensional, A is not diagonalizable.

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③ Construct an example of a matrix that is diagonalizable, is not diagonal, and is not invertible.

Diagonalizable means that an $n \times n$ matrix must produce eigenvalues whose eigenspaces have dimensions, when summed equal n.
Not diagonal means that it cannot be both upper triangular and lower triangular at the same time.
Not invertible means that the determinant must be 0.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow \det A = ad - bc = 0$$

Shows not invertible

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$$\det(A - \lambda I) = (a-\lambda)(d-\lambda) - bc = 0$$
$$\lambda^2 - a\lambda - d\lambda + \cancel{ad - bc}^{\,0} = 0$$
$$\lambda^2 - a\lambda - d\lambda = 0$$
$$\lambda(\lambda - a - d) = 0$$

$$\left.\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= a + d \neq 0 \end{aligned}\right\}$$ ← 2 distinct eigenvalue means 2 linearly independent eigenvectors which means diagonalizable.

$b \neq 0$ and $c \neq 0$ at same time. ← not diagonal

Given these constraints above, a possible construction is:

$$A = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$$

proven by: $\det A = 6(2) - 3(4) = 0$ ← not invertible

$$\det(A - \lambda I) = (6-\lambda)(2-\lambda) - 4(3) = 0$$
$$\lambda(\lambda - 8) = 0$$

$$\left.\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= 8 \end{aligned}\right\}$$ ← diagonalizable.

$b = 4,\ c = 3$ ← not diagonal.

④ Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ and $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ where $\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{b}_2 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$. Define $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$. Find $[T]_{\mathcal{B}}$

$$T(\vec{b}_1) = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$T(\vec{b}_2) = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

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Now, write $T(\vec{b})$ in terms of base $\mathcal{B}$:

$$B[X]_B = T(\vec{b}) \qquad B = \begin{bmatrix} 1 & 5 \\ 1 & 4 \end{bmatrix}$$
$$[X]_B = B^{-1} T(\vec{b})$$
$$B^{-1} = \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}$$

$$[\vec{X}_1]_B = \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$[\vec{X}_2]_B = \begin{bmatrix} -4 & 5 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

These form colums of $[T]_B$

$$[T]_B = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$

(5) Let $A = \begin{bmatrix} \sqrt{3} & 3 \\ -3 & \sqrt{3} \end{bmatrix}$. The transformation $\vec{x} \mapsto A\vec{x}$ is a composition of rotation and scaling.

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(a) give the angle of rotation and scale factor.

$a = \sqrt{3}$
$b = -3$

$$r = \sqrt{a^2 + b^2}$$
$$= \sqrt{(\sqrt{3})^2 + (-3)^2}$$
$$= \sqrt{3 + 9}$$
$$= \sqrt{12} = 2\sqrt{3}$$

pulling r out of A:

$$A = 2\sqrt{3} \begin{bmatrix} \frac{1}{2} & \frac{3}{2\sqrt{3}} \\ \frac{-3}{2\sqrt{3}} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2\sqrt{3} & 0 \\ 0 & 2\sqrt{3} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{3}{2\sqrt{3}} \\ \frac{-3}{2\sqrt{3}} & \frac{1}{2} \end{bmatrix}$$

$\cos\theta = \frac{1}{2}$ rotation $= -60°$
$\theta = 60°$ Scale $= 2\sqrt{3}$

(b) Find the eigenvalues and a basis for each eigenspace.

$$\det(A - \lambda I) = (\sqrt{3} - \lambda)(\sqrt{3} - \lambda) + 9 = 0$$
$$\lambda^2 - 2\sqrt{3}\lambda + 12 = 0$$
$$\lambda = \frac{2\sqrt{3} \pm \sqrt{12 - 4(12)}}{2} = \sqrt{3} \pm 3i$$

eigenspace for $\lambda_1 = \sqrt{3} + 3i$:

$A - \lambda I = \begin{bmatrix} 3i & 3 \\ -3 & 3i \end{bmatrix} \rightarrow$ Augmented $\rightarrow \left[\begin{array}{cc|c} 3i & 3 & 0 \\ -3 & 3i & 0 \end{array}\right]$

$\begin{array}{l} \frac{1}{3}R_1 \\ \frac{1}{3}R_2 \end{array} \left[\begin{array}{cc|c} i & 1 & 0 \\ -1 & i & 0 \end{array}\right]$

R1 has form: $ix_1 + x_2 = 0$

$x_1 = \frac{-x_2}{i}$ let $x_2 = i$

then $x_1 = -1$

basis for eigenspace: $\begin{bmatrix} +1 \\ -i \end{bmatrix}$

eigenspace for $\lambda_2 = \sqrt{3} - 3i$:

$A - \lambda I = \begin{bmatrix} -3i & 3 \\ -3 & -3i \end{bmatrix} \xrightarrow{\frac{1}{3}R_1 \ \frac{1}{3}R_2}$ Augmented $\rightarrow \left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$

R_2 has form: $-ix_1 + x_2 = 0$

$x_1 = \frac{x_2}{i}$ let $x_2 = i$

then $x_1 = 1$

basis for eigenspace: $\begin{bmatrix} 1 \\ i \end{bmatrix}$