Math 322 | Written Homework 7 | due **Tuesday April 4th at 6:00 p.m. in POT 741**

Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly. You may either turn in the assignment in class on Tuesday, or you may bring it to my office later in the day. Solutions will be posted on the course website Tuesday evening to help you prepare for the exam.

This assignment covers material in sections 6.1 through 6.4. The textbook is a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or possibly in the Mathskeller (depending on who is tutoring at the time).

1. Let $\mathbf{v}=\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, with $\mathbf{v}\neq\mathbf{0}$. Find a basis for the set H of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ that are orthogonal to $\mathbf{v}$.

2. Suppose we have a vector $\mathbf{u}\neq\mathbf{0}$ in $\mathbb{R}^n$. Let $L=\text{Span}\{\mathbf{u}\}$. Show that the mapping $\mathbf{x}\mapsto \text{proj}_L\,\mathbf{x}$ is a linear transformation.

3. Let $\mathbf{u}_1=\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2=\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $\mathbf{x}=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.
   a. Show that $\mathbf{x}$ is not in $\text{Span}\{\mathbf{u}_1,\mathbf{u}_2\}$.
   b. Show that $\mathbf{u}_1$ is orthogonal to $\mathbf{u}_2$ but $\mathbf{x}$ is not orthogonal to $\mathbf{u}_1$ or $\mathbf{u}_2$.
   c. Use $\mathbf{x}$ to construct a nonzero vector $\mathbf{v}$ in $\mathbb{R}^3$ that is orthogonal to $\mathbf{u}_1$ and $\mathbf{u}_2$.

4. Use the Gramm-Schmidt process to find an orthogonal basis for the column space of the matrix $A=\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$