Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly.

This assignment covers material in sections 6.5, 6.6, 4.1 and 4.2. The textbook is a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or possibly in the Mathskeller (depending on who is tutoring at the time).

1. Let $A=\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$.
   a. Show that the columns of A form an orthogonal basis for $\operatorname{Col} A$.
   b. Find the least-squares solution of $A\mathbf{x}=\mathbf{b}$ by finding $\hat{\mathbf{b}}$, the orthogonal projection of $\mathbf{b}$ onto $\operatorname{Col} A$, and then solving $A\mathbf{x}=\hat{\mathbf{b}}$.
   c. Now find the least-squares solution another way, by solving $A^TA\mathbf{x}=A^T\mathbf{b}$.

2. Suppose we collected data associating a variable x with a variable y, and found the data set (1, 7), (2, 4), (4, 3), (6, 1).
   a. Set up the design matrix, parameter vector and target vector to find the least-squares line of best fit, $y=b_0+b_1x$.
   b. Set up the design matrix, parameter vector and target vector to find the least-squares quadratic of best fit, $y=b_0+b_1x+b_2x^2$.
   c. Use technology to solve for the line and quadratic you set up in parts (a) and (b). (You can do these by hand if you want to, but that's not required). Then draw a graph that shows the data points and the line equation you found, and another graph with the data points and the quadratic you found. Which looks like it fits the data better?

3. Let $T:\mathbb{P}_2\to M_{2\times 2}$ be a linear transformation defined by $T\left(ax^2+bx+c\right)=\begin{bmatrix} a+b & a+c \\ a+c & a+b \end{bmatrix}$.
   a. Find $\ker T$
   b. Find the image of T (also called the range).