Please write up complete, clear solutions on your own paper. We will be looking for your reasoning and explanations, not just a correct answer. Please copy each question and write neatly.

This assignment covers material in Chapter 4. The textbook is a helpful reference for these. You can also get help via email (ewhitaker@uky.edu), via office hours (stop by or make an appointment) or possibly in the Mathskeller (depending on who is tutoring at the time).

1. Let $T : M_{2\times 2} \to M_{2\times 2}$ be a transformation defined by $T(A) = A + A^T$.

   a. Show that T is a linear transformation. *Hint*: begin by writing $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

   b. Find the kernel of T, a basis for the kernel of T, and the dimension of the kernel of T.

   c. Show that the image of T is the set of all matrices B in $M_{2\times 2}$ with the property $B^T = B$.

2. Let $\mathcal{B} = \{x + 2, 3x + 8\}$

   a. Show that $\mathcal{B}$ is a basis for $\mathbb{P}_1$, the space of polynomials of degree at most 1.

   b. Given $[p]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, find p.

   c. For $p = 7x + 10$, find the coordinate vector for p relative to the basis $\mathcal{B}$ (i.e., find $[p]_{\mathcal{B}}$).