Math 322 Review Problem 9 Solutions

Subspace or Not? Justify

(a) H in $M_{2\times 2}$ satisfying $\det(A)=0$

(i) is $\vec{0}$ in H? Yes: $\det\left(\begin{bmatrix}0&0\\0&0\end{bmatrix}\right)=0$

(ii) Scalar Mult? Yes: If A is any matrix in H and r is any real #, $\det(rA) = \underbrace{r^2\cdot\det(A)}_{\text{property of } 2\times 2 \text{ matrices}} = \underbrace{r^2(0)}_{\text{since } A \text{ is in } H} = 0$,

Thus rA is also in H.

(iii) Closed under addition? $\boxed{\text{NO}}$ ($\det(A+B) \neq \det A + \det B$).

$A_1 = \begin{bmatrix}1&1\\0&0\end{bmatrix}$ is in H, and $A_2 = \begin{bmatrix}0&0\\2&1\end{bmatrix}$ is in H,

but $\det(A_1+A_2) = \det\left(\begin{bmatrix}1&1\\2&1\end{bmatrix}\right) = 1-2=-1 \neq 0$,

so A_1+A_2 is not in H.

$\boxed{H \text{ is NOT a subspace.}}$

(b) H in $\mathbb{P}_3$ satisfying $p(5)=p(2)$.

(i) is $\vec{0}$ in H? Yes: the zero polynomial satisfies $p(5)=0$ and $p(2)=0$; thus $p(5)=p(2)$.

(ii) Closed under scalar mult? Yes: Let p be in H and r be any real number. Then

$(rp)(5) = \underbrace{r(p(5))}_{\text{can factor scalar out}} = \underbrace{r(p(2))}_{\text{since } p \text{ in } H} = (rp)(2)$.

Thus (rp) is in H.

(iii) closed under addition? Yes: Let p_1, p_2 be in H.

Then $(p_1+p_2)(5) = \underbrace{p_1(5)+p_2(5)}_{\text{poly. property}} = \underbrace{p_1(2)+p_2(2)}_{\text{since } p_1, p_2 \text{ in } H} = (p_1+p_2)(2)$,

So (p_1+p_2) is in H.

$\boxed{H \text{ \underline{is} a subspace.}}$