# MA 114 - Calculus II

## Final Exam

28 April 2008

Name:______________________________
Score:__________________ /100 Points

Instructions:

- This is a two hour exam. You may leave after you finish the exam and turn it in. However please do you talk about the exam until you are outside of ear shot of the classroom.
- You may not use any outside assistance on this exam. You may not use books, notebooks, other people's exams, or any other materials to cheat on this exam. There are some formulas listed on the back page that may or may not be useful over the course of the exam.
- You may not use a graphing calculator on this exam.
- The use of electronic equipment such as mp3 players, ipods, cell phones and other electronic devices during the exam is prohibited.
- If you are caught cheating on the exam, you will be given a 0 for a grade.
- Write clearly during the exam and fully erase or mark out anything you do not want graded.
- You must give exact answers and fully reduce fractions to receive full credit. Approximate and unreduced answers will receive only partial credit.
- **You must show all your work to receive full credit unless otherwise stated.**
- Have a great summer and enjoy Calculus III.

1. (5 points each) Calculate the following indefinite integrals. You must show all work.

   (a) $\int x \ln x dx$

   (b) $\int \cos^3(u) \sin^2(u) du$

(c) $\int \frac{\sqrt{9-x^2}}{x^2} dx$

2. (10 points) Evaluate the integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$.

3. (3 points each) Answer each of the following questions either True or False. You must write the entire word to get full credit. You do not need to show any work on this problem.

(a) If $0 < a < b$, then $\ln a < \ln b$.

(b) If $f(x) \leq g(x)$ and $\int_0^\infty g(x)dx$ diverges, then $\int_0^\infty f(x)dx$ also diverges.

(c) If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $d^2y/dx^2 = (d^2y/dt^2)/(d^2x/dt^2)$.

(d) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent.

(e) $\lim_{x\to\infty} \frac{\ln x + 2x}{x} = 0$

4. (10 points) Determine if the following series is convergent or divergent:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}.$$

You must state which test you are using and verify all of the conditions required to apply it.

5. (10 points) Find the radius of convergence and interval of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}.$$

6. We all love the function $g(x) = \sin x$.

(a) (3 points) What is the Maclaurin series for $g(x)$? You do not need to show your work for this part.

(b) (7 points) Using part $(a)$, determine the Taylor series for

$$f(x) = (x - \pi)^2 \sin(x)$$

centered at $a = \pi$ and find its radius of convergence.

7. (10 points) The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

is known to converge. Determine the limit of the series using any means necessary.

8. (10 points) Use calculus to prove **any one** of the following claims about a circle with radius $r > 0$. Label clearly which statement is being proved.

   (a) The area of the circle is $\pi r^2$.

   (b) The circumference of the circle is $2\pi r$

9. (10 points) Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

10. Extra Credit: The **Fibonacci sequence** $\{f_n\}$ is defined recursively by the conditions:

$$f_1 = 1,\ f_2 = 1,\ f_n = f_{n-1} + f_{n-2}, \text{ for } n \geq 3.$$

(a) (3 points) Determine the Fibonacci numbers $f_1$ through $f_{10}$.

(b) (2 points) Suppose rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. Prove if we start with one pair of pet rabbits, then in the $n^{th}$ month we will have $f_n$ pairs of rabbits. (Note that giving specific examples is not a proof).

## Some Formulas

i. $L = \int_{\alpha}^{\beta} \sqrt{(dx/dt)^2 + (dy/dt)^2}dt$

ii. $L = \int_{\alpha}^{\beta} \sqrt{r^2 + (dr/d\theta)^2}d\theta$

iii. $A = \int_{a}^{b} \frac{1}{2}r^2 d\theta$

iv. $a_n = \frac{f^{(n)}(a)}{n!}$