

Review 2:

This review is meant as a general overview of SOME of the topics covered in class up to date. The test questions will not only cover this material but will also cover sections 8.2-8.5, 8.7-8.8, 12.1-12.3. You should know all definitions, theorems, and techniques outlined in the text, as well as be comfortable with the properties and examples throughout the above sections. Below I provide some sample problems that cover material from class. I am in no way promising any of these problems will be on the test. They are solely for practice. Other good sample problems can be found in ‘suggested problems’ sections of your homework assignments, lecture notes, and class handouts, as well as the before mentioned places.

1. Evaluate the following integrals.

•

$$\int \tan^3(x+1) \sec^3(x+1) dx$$

•

$$\int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta$$

•

$$\int \frac{\sqrt{x^2-9^2}}{x^4} dx$$

•

$$\int_0^1 x\sqrt{x^2+4} dx$$

2. Evaluate the following integrals using partial fractions.

•

$$\int \frac{1}{(x-2)(x+3)} dx$$

•

$$\int \frac{r^2}{r+5} dr$$

•

$$\int \frac{ds}{s^2(s-1)}$$

•

$$\int \frac{x-1}{(x+4)(x^2+1)} dx$$

3. Use a) the Trapezoid Rule, b) the midpoint rule, c) Simpson’s Rule to approximate each of the integrals below for the specified n . Then find the error estimate of each.

- $\int_0^4 x^5 - x^3 dx$ for $n = 4$
- $\int_0^{1/2} \sin(x^2) dx$, for $n = 4$

4. Determine if the following integrals are convergent or divergent. Evaluate those that are convergent.

•

$$\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw$$

•

$$\int_0^{\infty} s e^{-5s} ds$$

•

$$\int_{-1}^0 \frac{1}{x^2} dx$$

5. Use the comparison theorem to determine if the integral is convergent or divergent:

$$\int_0^{\pi/2} \frac{dx}{x \sin x}$$

6. Determine if whether the sequence converges or diverges. If it converges, find the limit.

• $a_n = \frac{n+1}{3n-1}$

• $a_n = \frac{3^n}{\pi^{n-1}}$

• $a_n = \frac{\cos^2 n}{2^n}$

7. Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

• $a_n = \frac{1}{5^n}$

• $a_n = \frac{n}{n^3+1}$

• $a_n = n + \frac{1}{n}$

8. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

•

$$\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^{n-1}$$

•

$$\sum_{k=2}^{\infty} \frac{k^2}{k^2 - 1}$$

•

$$\sum_{t=1}^{\infty} \sqrt[t]{2}$$

9. If $\sum a_n$ and $\sum b_n$ are both divergent, is $\sum(a_n + b_n)$ necessarily divergent?

10. If $\sum a_n$ is convergent and $\sum b_n$ is divergent, prove that the series $\sum(a_n + b_n)$ is divergent. (hint: prove by contradiction.)

11. All suggested homework problems and worksheet problems.

12. Read the book.