

## WorkSheet7

1. Evaluate the following trigonometric integrals.

- $$\int \sqrt{1 - 4x^2} dx$$

- $$\int \frac{\sqrt{1 + x^2}}{x} dx$$

2. Evaluate the integral

$$\int \frac{t dt}{\sqrt{t^2 - 4}}$$

first by  $u$ -substitution and then by trigonometric substitution.

3. Find the area enclosed by the ellipse:

$$\frac{s^2}{16} + \frac{t^2}{25} = 1.$$

4. Evaluate the following integrals using partial fractions.

- $$\int \frac{dx}{x^3 - x^2 - 2x}$$

- $$\int \frac{t^3 + 1}{t^3 - 1} dt$$

- $$\int \frac{x + 2}{x(x - 1)^2} dx$$

5. Evaluate the integral:

$$\int \frac{dx}{3 \cos x - 4 \sin x}.$$

This is a hard problem without the insight of the German mathematician Karl Weierstrass (1815-1897) who realized you can use the substitution  $u = \tan \frac{x}{2}$  for  $-\pi < x < \pi$ . First solve this equation in  $u$  and  $x$  for  $x$ . Then determine  $dx$ ,  $\sin x$ , and  $\cos x$ . Finally make your substitutions and solve by partial fractions for the variable  $u$  before substituting back in for  $x$ .