MA 322 - Matrix Algebra

Exam 3

16 April 2009

Name:____________________________

Score:_________________ /100 Points

Instructions:

- You may not use any outside assistance on this exam. You may not use books, notebooks, other people's exams, or any other materials to cheat on this exam.
- You may not use a graphing calculator on this exam.
- The use of electronic equipment such as mp3 players, ipods, cell phones and other electronic devices during the exam is prohibited.
- If you are caught cheating on the exam, you will be given a 0 for a grade.
- Write clearly during the exam and fully erase or mark out anything you do not want graded.
- You must give exact answers and fully reduce fractions to receive full credit. Approximate and unreduced answers will receive only partial credit.
- **You must show all your work to receive full credit unless otherwise stated.**

1. (5 points each) Determine if the following sets of vectors are vector spaces or not. Justify your answer by either showing why the set is a vector space or give a counter example to why it is not.

   (a)

$$\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 2 \right\}$$

   (b)

$$\left\{ \begin{bmatrix} d - 6f \\ d \\ f \end{bmatrix} : d,\ f \in \mathbb{R} \right\}$$

2. Let W be the subspace spanned by the vectors v_1 and v_2 and let y be the vector all defined below.

$$y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \; v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

(a) (10 Points) Find the closest point to y in the subspace W.

(b) (10 Points) Write y as the sum of a vector in W and a vector orthogonal to W.

3. Let $v = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$.

(a) (10 Points) Find a non-zero vector u that is orthogonal to v.

(b) (10 Points) State the Pythagorean theorem in terms of *norms* and show it holds true for u and v.

(c) (10 Points) Normalize the vectors u and v from part 3a

4. (10 Points) Let $\mathbb{P}_2$ be the set of all polynomials of at most degree 2 with real coefficients. Prove that the following set is a basis for $\mathbb{P}_2$.

$$\{p_1(t) = 3,\ p_2(t) = t - 6,\ p_3(t) = t^2 + 3t + 12\}$$

5. Consider the following matrix A and the REF form of A below:

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix} \sim \begin{bmatrix} -1 & 6 & 6 \\ 0 & 10 & 21 \\ 0 & 0 & 18/5 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) (10 Points) What is a basis for $Col(A)$?

(b) (10 Points) Determine an orthogonal basis for $Col(A)$.

(c) (10 Points) Is your orthogonal basis orthonormal? Justify your answer a calculation.

6. Extra Credit: (5 Points) Describe all the least-squares solutions of the equation $Ax = b$ where:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}.$$