3 April 2009

Review 3:

This review is meant as a general overview of SOME of the topics covered in class up to date. The test questions will not only cover this material but will also cover sections 6.1-6.5 and 4.1-4.3. You should know all definitions and techniques outlined in the text, and be comfortable with the properties and examples throughout the above sections as well as know how to solve the exercises and homework problems. Below I provide some sample problems that cover material from class. I am in no way promising any of these problems will be on the test. They are solely for practice.

1. Prove that the set V of all real valued functions defined on $\mathbb{R}$ is a vector space with vector addition and scalar multiplication being defined by:

$$(f+g)(t) = f(t) + g(t),\ (cf)(t) = cf(t)$$

for f, $g \in V$ and c a scalar in $\mathbb{R}$.

2. Determine a non-standard basis for $\mathbb{P}_3$ and show that your set is in fact a basis.

3. Consider the following matrix A below:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}.$$

   (a) What is a basis for $Col(A)$?

   (b) Determine an orthogonal basis for $Col(A)$.

4. Compute the least-squares solution of $Ax = b$ where:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix},\ b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}.$$

5. Determine if the following sets of vectors are vector spaces or not. Justify your answer by either showing why the set is a vector space or give a counter example to why it is not.

   (a)
$$\left\{ \begin{bmatrix} b-5d \\ 2b \\ 3d \\ d \end{bmatrix} : b,\ d \in \mathbb{R} \right\}$$

   (b)
$$\left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\}$$

6. Let $v = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

   (a) Find a non-zero vector u that is orthogonal to v.

   (b) State the Pythagorean theorem in terms of *norms* and show it holds true for u and v.

   (c) Normalize the vectors u and v from part 6a.