Name: ____ Date: 26 February 2009

QUIZ 4

Consider the matrix:

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. (4 points) Find a basis for Col A.

   We know that the pivot columns of a matrix form a basis for its column space. The pivot columns of A are the same as the pivot column to the equivalent matrix. Therefore the pivot columns of A are columns a_1 and a_2. Hence a basis of Col A is given by:

$$\left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\}.$$

2. (4 points) Find a basis for Nul A.

   The nullspace of A is the set of solutions to the equation $Ax = 0$. Thus we form the augmented matrix $[A\ 0]$ and see:

$$[A\ 0] = \begin{bmatrix} 4 & 5 & 9 & -2 & 0 \\ 6 & 5 & 1 & 12 & 0 \\ 3 & 4 & 8 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 7 & 0 \\ 0 & 1 & 5 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   This equivalent matrix corresponds to the system of equations

$$\begin{aligned} x_1 &= 4x_3 - 7x_4 \\ x_2 &= -5x_3 + 6x_4. \end{aligned}$$

   A general solution then looks like

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4x_3 - 7x_4 \\ -5x_3 + 6x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix}.$$

   Thus a basis for Nul A is given by:

$$\left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

3. (2 points) What is rank A? Be sure to justify your answer.

   The rank of a matrix A is the dimension of the column space. Since the basis for Col A has 2 vectors in it, $rank A = 2$.