

MA 111 Review for Exam 1

Exam 1 (given in class on Wednesday, Sept. 30) will cover Chapters 1 and 2.

You should be familiar with the following key ideas:

1. Preference schedules: Be able to create and read/interpret one.
2. Be able to declare a winner of an election using four voting methods:
 - (a) the Plurality method
 - (b) the Borda Count method
 - (c) the Plurality-with-Elimination (i.e. Instant Runoff) method
 - (d) the Method of Pairwise Comparisons
3. Understand four fairness criteria:
 - (a) the Majority Criterion
 - (b) the Condorcet Criterion
 - (c) the Monotonicity Criterion
 - (d) the Independence-of-Irrelevant-Alternatives Criterion (IIA)
4. Using a given example, be able to explain how that example shows that one of the voting methods violates one of the fairness criteria.
5. Be able to extend the four voting methods to produce a ranking of the candidates. You do not need to know the recursive rankings.
6. Be able to...
 - (a) ... count the number of Borda points awarded per ballot and in an entire election;
 - (b) ... count the number of pairwise comparisons in a given election.
7. Understand the notation for a weighted voting system, including conditions on the quota.
8. Be able to create the $[q : w_1, \dots, w_N]$ notation and to read information from such notation.
9. Be able to find the total number of possible sequential coalitions.
10. Be able to compute the Banzhaf power index for each player in a weighted voting system.
11. Be able to compute the Shapley-Shubik power index for each player in a weighted voting system.
12. Be able to identify dictators, dummies, and players with veto powers in a system.

Practice Problems

Use exercises in the text to supplement these for extra practice. (The odd-numbered problems have solutions in the back of the book so that you can check your answers.) Review the homework exercises, practice quizzes, and the examples in the text.

1. Consider the following preference schedule:

Number of Voters	11	10	4	7
1st choice	A	D	B	C
2nd choice	B	C	A	B
3rd choice	C	B	C	D
4th choice	D	A	D	A

- (a) How many first-place votes does a candidate need to be a majority candidate? Is there a majority candidate?
- (b) How many pairwise comparisons must be done to carry out the Method of Pairwise Comparisons? Who is the winner under the Method of Pairwise Comparisons?
- (c) If we use the Borda Count Method, what is the total number of points awarded to all the candidates?
- (d) Find the winner by the Plurality Method.
- (e) Rank the candidates using the extended Borda Count Method.

2. Consider the following preference schedule:

Number of Voters	10	6	5	4	2
1st choice	A	B	B	C	D
2nd choice	C	D	C	A	C
3rd choice	B	C	A	D	B
4th choice	D	A	D	B	A

- (a) Is there a Condorcet candidate in this election? Is there a majority candidate?
- (b) Find the winner using the Plurality-with-Elimination Method.
- (c) Suppose that **B** drops out of the race. Find the winner of the recount when B is removed using the plurality-with-elimination method.
- (d) You have just shown that plurality-with-elimination violates several fairness criteria. Which ones? Explain your answer.

3. An election is to be decided using the Borda count method. There are four candidates (A,B,C,D) in this election.
- How many points are given out by one ballot?
 - If there are 110 voters in the election, what is the total number of points given out to the candidates?
 - If candidate A gets 320 points, candidate B gets 290 points, and candidate C gets 180 points, how many points did candidate D get?
 - Who is the Borda winner of this election?
4. Answer the following questions about the preference schedule given below.

# of Voters	5	3	2	4	1
1st choice	A	A	B	C	C
2nd choice	B	C	A	A	B
3rd choice	C	B	C	B	A

- How many votes were cast in the election?
 - How many votes are needed for a majority?
 - Who is the plurality winner?
 - Does the plurality winner have a majority?
 - In a Borda count, how many points does candidate C get?
 - Between candidates B and C, who is preferred head-to-head?
 - In the first round of plurality with elimination, is there a winning candidate? If not, which candidate is eliminated first?
5. Explain why the method of pairwise comparisons satisfies the Condorcet criterion.
6. Explain why the method of pairwise comparisons satisfies the majority condition.
7. Consider the weighted voting system $[q : 10, 8, 7, 5, 4, 4, 3]$.
- What is the smallest possible value of q ?
 - Are there any permissible values of q that will result in a dictator?
 - Find all the values of q so that the only winning coalition is the grand coalition.
8. Consider the weighted voting system $[9 : 5, 5, 4, 2, 1]$.
- How many players are in this system?
 - Find the total number of votes.
 - Find the Banzhaf power distribution for such a system.
 - How many different sequential coalitions of all players would we have in such a system?

9. Find the Shapley-Shubik power distribution for the following weighted voting systems:
- (a) $[8 : 8, 5, 1]$ Are there any dictators? Any dummies? Any players with veto power?
 - (b) $[8 : 6, 5, 3]$ Are there any dictators? Any dummies? Any players with veto power?
10. In a weighted voting system with 5 players, we have only the following winning coalitions:

$$\begin{aligned} & \{P_1, P_2, P_3\} \\ & \{P_1, P_2, P_3, P_4\} \\ & \{P_1, P_2, P_3, P_5\} \\ & \{P_1, P_2, P_4, P_5\} \\ & \{P_1, P_2, P_3, P_4, P_5\} \end{aligned}$$

Find the Banzhaf power distribution for this voting system.

11. Consider the weighted voting system $[q : 10, 6, 5, 4, 2]$.
- (a) What is the smallest value that the quota q can take?
 - (b) What is the largest value that the quota q can take?
 - (c) What is the minimum value of the quota if at least two-thirds of the votes are required to pass a motion?
 - (d) What is the minimum value of the quota if at least three-fourths of the votes are required to pass a motion?
12. Consider the weighted voting system $[q : 5, 3, 1]$. Find the Banzhaf power distribution of this weighted voting system when
- (a) $q = 5$
 - (b) $q = 7$
 - (c) $q = 9$
13. Consider the weighted voting system $[q : 5, 3, 1]$. Find the Shapley-Shubik power distribution of this weighted voting system when
- (a) $q = 5$
 - (b) $q = 7$
 - (c) $q = 9$