Exam 1 - Version 1 MA 111

You may not use your book, notes, cell phone, or graphing calculators during the exam. Scientific and four-function calculators are allowed. If you have any questions during the exam, please raise your hand, and I will come to you. Please organize your work neatly, and provide justifications for your answers. **Unsupported answers may not receive credit**. The number in the parentheses next to each problem is how many points that problem is worth.

September 30, 2009

Name: _____

Problem	Points	Value
1		10
2		9
3		2
4		6
5		5
6		12
7		11
8		3
9 (Bonus)		0
Total		58

1. (10pts) For this question, use the following preference schedule:

Number of Voters	6	2	3
1^{st} choice	A	B	C
2^{nd} choice	B	C	D
3^{rd} choice	C	D	B
4^{th} choice	D	A	A

(a) How many people voted in this election?

(b) How many first-place votes are needed for a majority?

(c) Is there a majority candidate in this election? If so, who?

(d) Give a **ranking** of the candidates using the extended Borda Count method. Be sure to show all of your Borda totals in your work.

1^{st} place: _____

2^{nd} place: _____

3^{rd} place: _____

4^{th} place: _____

(e) Your responses above should show that the Borda Count method violates a fairness criterion. Which one? Justify your response.

2. (9pts) For this question, use the following preference schedule:

Number of Voters	10	12	8	4
1^{st} choice	B	C	A	A
2^{nd} choice	C	A	B	C
3^{rd} choice	A	B	C	B

(a) How many votes are needed to have a majority in this election?

(b) Find the winner of the election using the plurality-with-elimination method. Be sure to clearly show your work.

(c) Suppose that the 4 voters in the last column decide to flip their first two preferences, making their first choice C and their second choice A. Who is the winner in this situation using the plurality-with-elimination method? Be sure to clearly show your work.

(d) Your responses above should show that the plurality-with-elimination method violates one of the fairness criteria. Which one? Justify your answer.

3. (2pts) Suppose candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison. Which one of our fairness criteria says that candidate X should be the winner of the election?

4. (6pts) For this question, use the following preference schedule:

Number of Voters	5	4	2
1^{st} choice	A	B	D
2^{nd} choice	B	D	A
3^{rd} choice	C	C	C
4^{th} choice	D	A	B

(a) Find the winner of this election using the plurality method.

(b) Suppose that candidate B drops out of the race. Find the new winner under the plurality method.

(c) Your responses above should show that the plurality method violates one of the fairness criteria. Which one? Justify your answer.

5. (5pts) Consider the weighted voting system [q: 10, 6, 5, 4, 4, 3, 2].

(a) How many players are in this system? _____

(b) What is the total number of votes in this system? _____

(c) What is the weight of P_3? _____

(d) What is the smallest value that the quota q can take? _____

(e) What is the largest value that the quota q can take? _____

6. (12pts) Find the Shapley-Shubik power distribution of the following weighted voting systems. Be sure to clearly justify your answers.

   (a) $[5 : 5, 2, 1]$

   (b) $[9 : 5, 4, 3]$

7. (11pts) In a weighted voting system with five players the winning coalitions are as follows:

$$\{P_1, P_2, P_3\}$$

$$\{P_1, P_2, P_4\}$$

$$\{P_1, P_2, P_3, P_4\}$$

$$\{P_1, P_2, P_3, P_5\}$$

$$\{P_1, P_2, P_4, P_5\}$$

$$\{P_1, P_2, P_3, P_4, P_5\}$$

   (a) In each of the above winning coalitions, clearly underline all of the critical players in each coalition.

   (b) Find the Banzhaf power distribution of the above weighted voting system.

   (c) Is there a dictator? If so, who? _____

   (d) Are there any dummies? If so, who? _____

8. (3pts) Consider a weighted voting system with 5 players, P_1, P_2, P_3, P_4, P_5.

   (a) Find the total number of sequential coalitions of all players in this weighted voting system.

   (b) Find the total number of sequential coalitions of all players in this weighted voting system where P_2 is listed first.

   (c) Find the total number of sequential coalitions of all players in this weighted voting system where P_5 is listed first and P_2 is listed third.

9. **Bonus:** (3pts) In an election with 7 candidates, what is the maximum number of columns possible in the preference schedule.