

MA 111: Review for Exam 2

Exam 2 will cover material from Chapters 5 and 6. Use this review sheet, past homework exercises, practice quizzes, and examples from the text to help you study. You should be familiar with the following concepts:

Chapter 5:

- Know what a graph is and how to use graphs to model geographic relationships. Know how to describe the vertex set and edge set by looking at a graph, and know how to draw the picture by studying the vertex set and edge set.
- Recall the following vocabulary related to graphs: adjacent edges; adjacent vertices; degree of a vertex; path; circuit; connected graph; bridge.
- Be able to recognize an **Euler path** or an **Euler circuit** in a graph (that is, know the definition of Euler path/circuit). Alternatively, be able to say why a given path/circuit is not an Euler path/circuit.
- Use **Euler's Theorems** to decide whether or not a given graph has an Euler path, an Euler circuit, or neither.
- Be able to state and use **Euler's Sum of Degrees Theorem**.
- Effectively implement **Fleury's Algorithm** to find an Euler path or circuit in a given graph.
- Be able to **eulerize** or **semi-eulerize** a graph, so that you can then find an Euler circuit or Euler path.
- Be able to find an **optimal eulerization** or an **optimal semi-eulerization** of a graph.

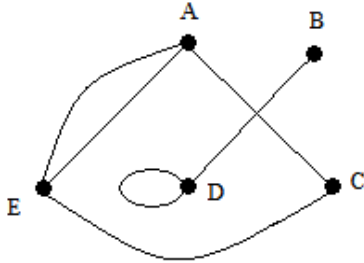
Chapter 6:

- Know the definitions of **Hamilton path** and **Hamilton circuit**. Be able to say why a given path/circuit is or is not a Hamilton path/circuit.
- Understand the definition of a complete graph K_N . Be able to compute the number of edges, the degree of every vertex, and the number of different Hamilton circuits in K_N for any N .
- Know the definition of a weighted graph.
- Understand the goal of solving a **traveling salesman problem**.
- Be able to implement the following **four algorithms** for finding Hamilton circuits in complete graphs:
 - a) Brute-Force
 - b) Nearest-Neighbor
 - c) Repetitive Nearest-Neighbor
 - d) Cheapest Link
- Understand the pros and cons of using any of the four algorithms above.

Practice Problems:

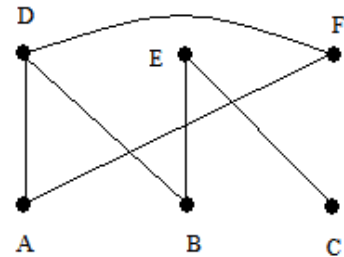
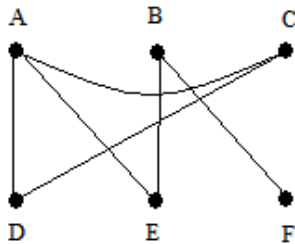
As you work through these exercises, think of related questions that may be on the exam. How could a clever instructor like me twist these questions into something different?

- 1) List the vertex set V and the edge set E for the following graph:



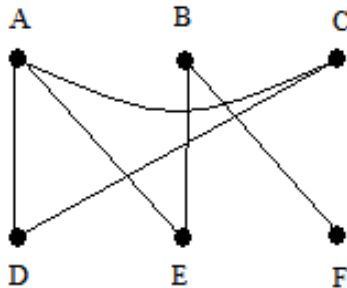
- 2) List the edge set E of the complete graph K_5 , consisting of the five vertices $\{A, B, C, D, E\}$.

- 3) Explain why the following two figures do not represent the same graph:

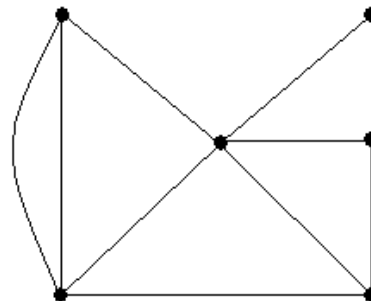


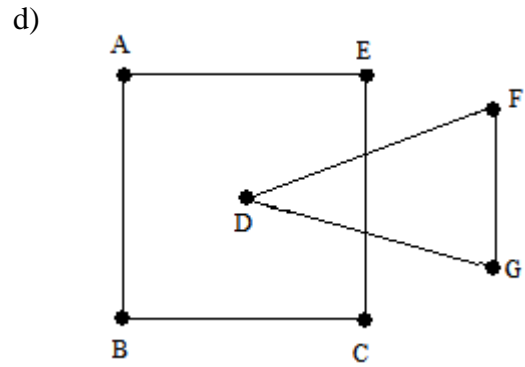
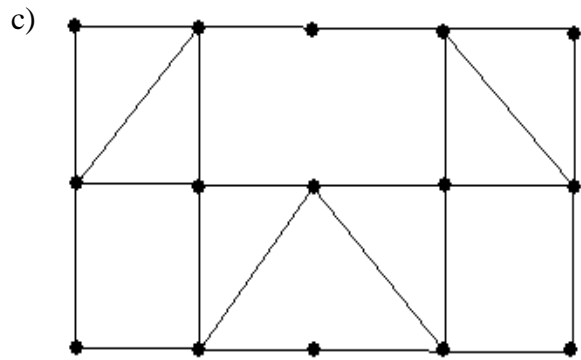
- 4) For the following graphs, determine whether the graph has an Euler path, an Euler circuit, or neither. Justify your responses.

a)

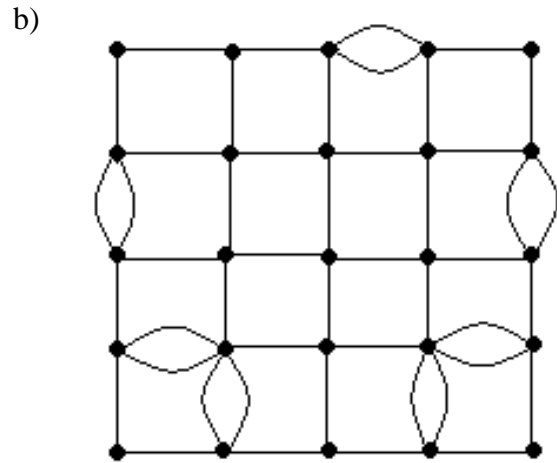
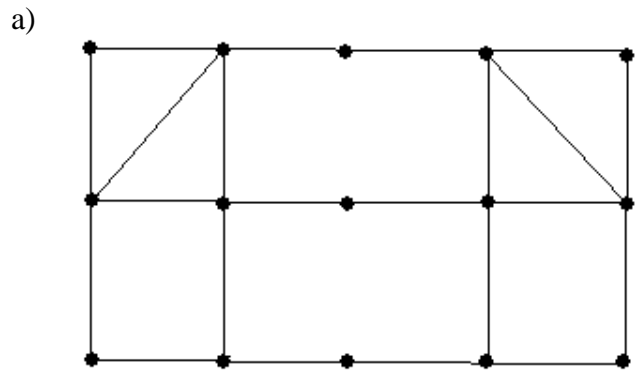


b)

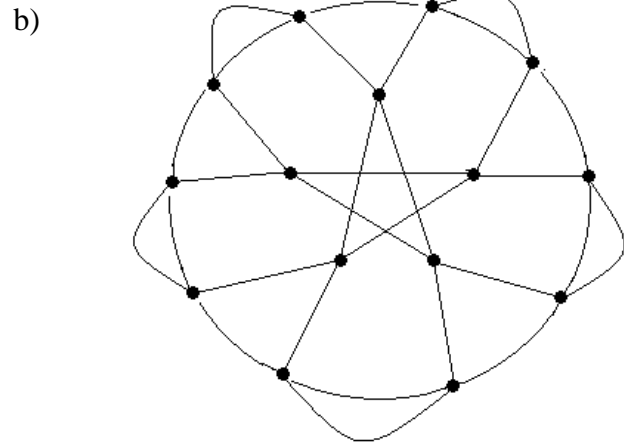
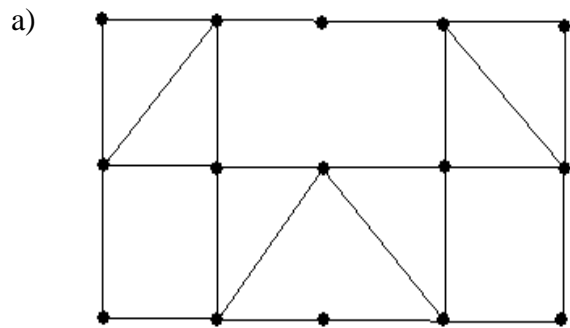




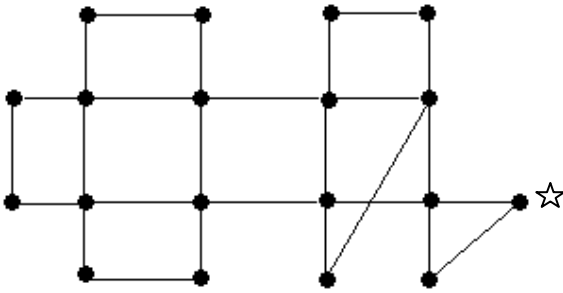
5) Use Fleury's Algorithm to find an Euler path in the following graphs:



6) Use Fleury's Algorithm to find an Euler circuit in the following graphs:

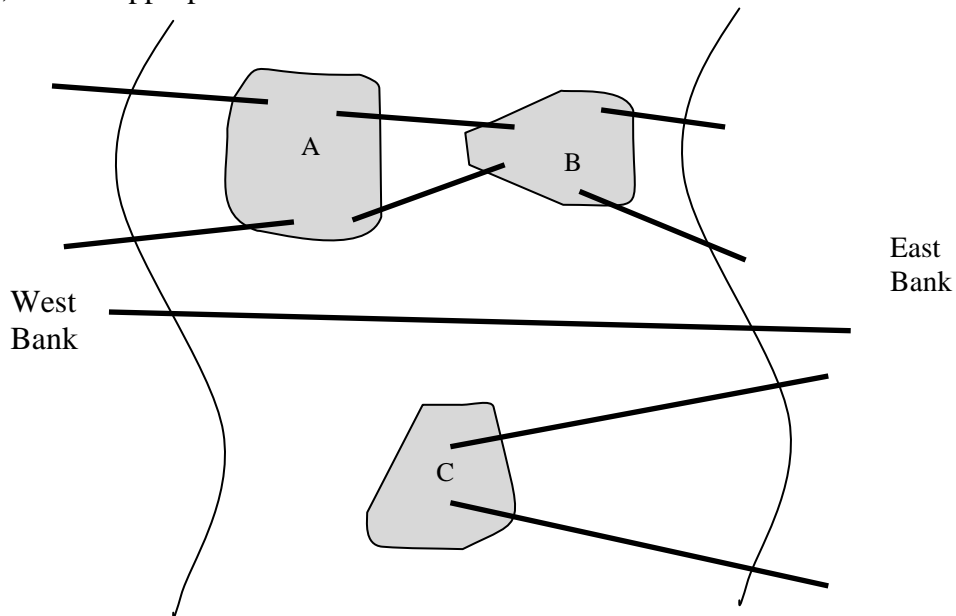


c) Start and end at the starred vertex:



7) Below is a sketch of a town. A, B, and C represent islands in the middle of the town's river. There are nine bridges joining the islands and the two banks of the river; the bridges are represented by bold lines.

Is it possible for the town's tourism board to design a bus tour which crosses each bridge once and only once? If so, will the tour begin and end at the same place, or not? If such a bus tour is possible, find an appropriate route.

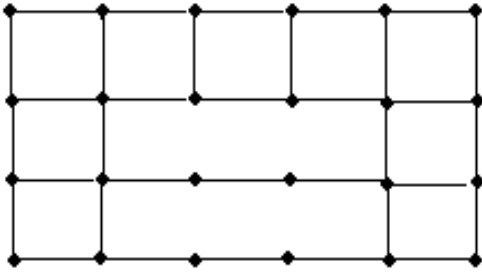


8) Recall the goals and rules behind eulerizing graphs:

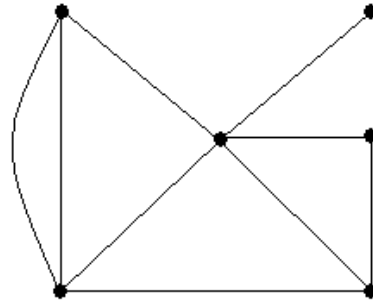
- In order to make the graph into one which contains an Euler circuit, you must turn all odd vertices into even vertices and make sure that all even vertices remain even.
- The easiest way to accomplish the above is to add in new edges between odd vertices in a pairwise fashion. But there are complications...
- You cannot create a completely new edge in a graph. Instead, the edges you add must be duplicates of edges already in the original graph. If you break this rule, then in a specific example you may have a bus driving through the middle of a city block. Avoid this.

Find an eulerization of each of the following graphs:

a)



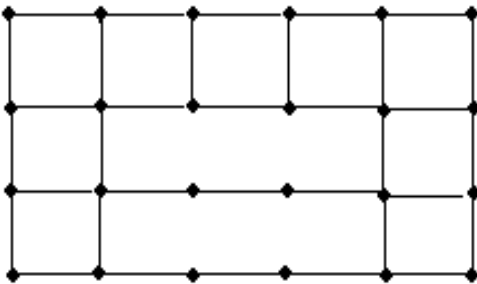
b)



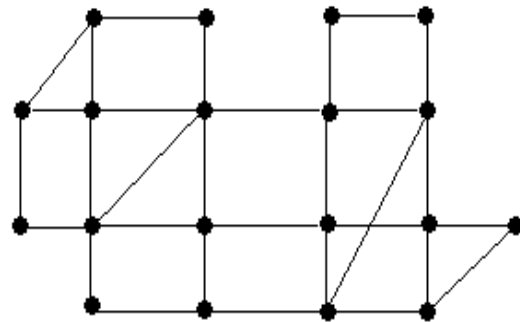
9) Recall that the goals and rules behind semi-eulerization are almost the same as those above. However, to make the graph into one which contains an Euler path, you must now leave two of the vertices as odd vertices, while making all others into even vertices.

Find a semi-eulerization of each of the following graphs.

a)



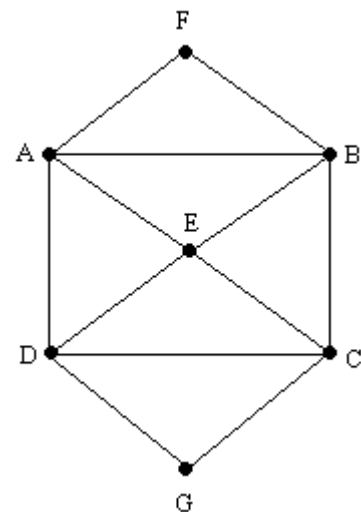
b)



10) Suppose you have a connected graph G , consisting of N vertices, in which every vertex has the same degree. If N is an odd number, explain why G must have an Euler circuit. (Hint: Euler's Sum of Degrees Theorem)

11) Consider the graph to the right. Which of the following lists of vertices describe a Hamilton circuit? For those that do not describe Hamilton circuits, why not?

- a) A, F, B, C, G, D
- b) A, F, B, C, G, D, A
- c) B, E, A, F, D, G, C, B
- d) B, F, A, E, D, G, C, B



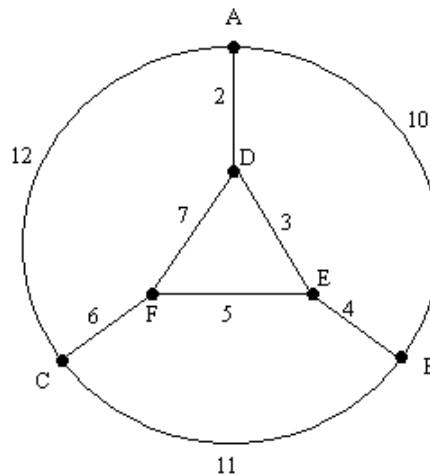
12) For the weighted graph shown to the right,

a) Find the total weight of the path:

C, F, D, A, B, E

b) Find a Hamilton path that starts at

A and ends at C, and give its total weight.



13) a) How many edges are in K_{10} , the complete graph with 10 vertices?

b) How many distinct Hamilton circuits are contained in K_{10} ?

c) Pick any vertex in K_{10} . What is the degree of the vertex?

14) You work at FedEx, and the company has just decided to add a new truck route that will cover 9 cities in China. (You can assume that each city is connected to each other city.)

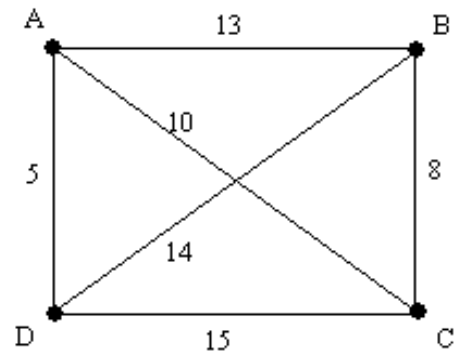
You want to design the route in the most efficient way, so that the truck will never have to visit a city more than once, except for Beijing, where the FedEx headquarters is. Also for efficiency, you want the total distance traveled to be as small as possible.

Suppose that for one possible route, your computer can compute the total distance traveled in half of a second. If your boss tells you to give him the *optimal* route by the end of the day (within 8 hours), can you do it? If so, say how many hours it will take you to complete the project. If not, say why not.

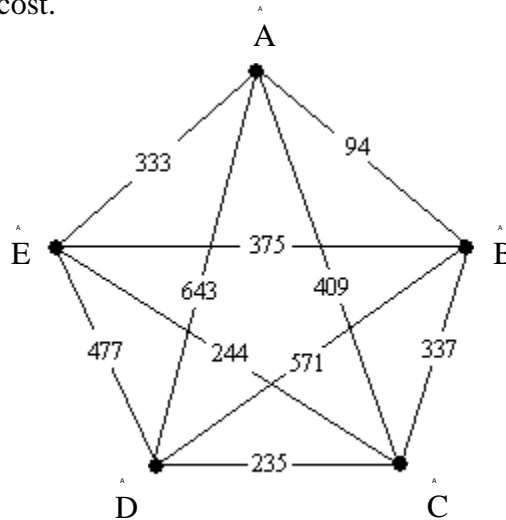
(Hint: We are considering a weighted graph of K_9 , where the weights are representing distances between cities.)

15) Consider the following weighted graph:

- Find the nearest-neighbor tour starting at vertex C.
- Use the repetitive nearest-neighbor algorithm to find a near-optimal Hamilton circuit starting at C and its cost.
- Is your answer to (b) optimal? Justify your response.



16) For the following graph, use the cheapest-link algorithm to find a near-optimal Hamilton circuit starting at D and its cost.



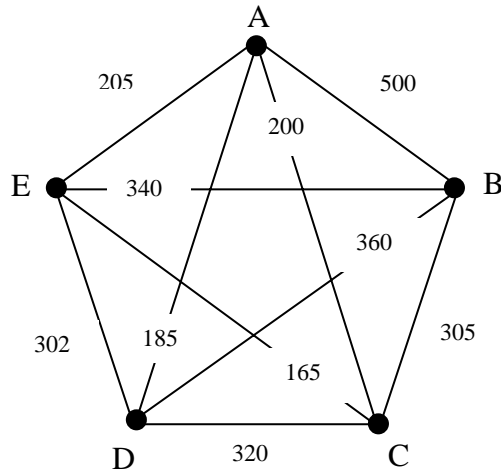
17) Suppose you are renting a car in Detroit and need to visit all of the cities listed in the mileage chart below and end up back in Detroit. The car company charges a flat rate of 49 cents per mile. Use the repetitive nearest-neighbor algorithm to find a near-optimal route, and give the total dollar cost of the trip.

	Detroit	Lansing	Grand Rapids	Flint	Cheboygan	Kalamazoo
Detroit	*	90	158	68	280	140
Lansing	90	*	68	56	221	75
Grand Rapids	158	68	*	114	233	51
Flint	68	56	114	*	215	131
Cheboygan	280	221	233	215	*	277
Kalamazoo	140	75	51	131	277	*

18) Consider the weighted graph shown below.

a) Find an optimal tour using the brute-force algorithm, and give its cost. (Hint: Use mirror circuits to cut your work in half.)

b) Find an optimal tour that starts with the edge EB, and give its cost.



19) Consider the weighted graph shown above. Find the cheapest-link tour that starts at E, and give its cost.

20) Explain why a graph that has a bridge cannot have a Hamilton circuit.

21) Explain why the cheapest edge in any graph is always part of the Hamilton circuit obtained using the nearest-neighbor algorithm.