

MA 111 Review for Exam 3

Exam 3 will cover Chapters 9 and 3.

Chapter 9:

- Know the recursive definition of the Fibonacci numbers, and be able to generate them recursively.
- Be able to do computations involving Fibonacci numbers and arithmetic operations on the subscripts.
- Know that there is an explicit formula that can be used to generate the Fibonacci numbers. You do not need to memorize this formula, but you should be able to say why it is useful.
- Be able to solve quadratic equations using the quadratic formula.
- You do not need to memorize the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Know that the golden ratio, ϕ , satisfies the golden property, that is, $\phi^2 = \phi + 1$.
- Be familiar with the relationships between ϕ and the Fibonacci numbers. Specifically, $\phi^N = F_N\phi + F_{N-1}$ and $F_N/F_{N-1} \approx \phi$.
- Know what it means for two triangles, rectangles, squares, circles, or circular rings to be similar. Be able to use similarity to solve for unknown quantities.
- Know the definition of a gnomon, and be able to use the idea of gnomons to solve for unknown quantities.

Chapter 3:

- Understand and be able to explain the basic elements and assumptions of fair-division games.
- Know what it means for a portion to be a fair share to a player.
- Be able to compute the value of a given portion and decide if it is a fair share or not.
- Be able to find a fair division (if one exists), or explain why it doesn't exist.
- Understand the difference between continuous and discrete goods.
- Understand how the Divider-Chooser method works for two players.
- Be able to carry out the Divider-Chooser method given players' value systems.
- Understand how the Lone-Divider and Lone-Chooser methods work for three players.
- Be able to carry out each method given players' value systems.
- Understand how each method works for four or more players.
- Understand how the method of sealed bids works for two or more players.
- Be able to carry out the method of sealed bids.
- Understand how the method of markers works.
- Be able to carry out the method of markers (both placing the markers and allocating the items).

Practice Problems

Use exercises in the text to supplement these for extra practice. (The odd-numbered problems have solutions in the back of the book so that you can check your answers.) Review the homework exercises, practice problems, and the examples in the text.

1. Describe in words what each of the expressions represents.

(a) $F_{2N} - 3$ (b) F_{2N-3} (c) $2F_N - 3$ (d) $2F_{N-3}$

2. Compute the value of each of the following.

(a) F_7/F_4 (c) F_{3+6+2} (e) F_{13-6} (g) $F_5 \times F_8$
 (b) $F_{9/3}$ (d) $F_3 + F_6 + F_2$ (f) $F_{12} - F_8$ (h) $F_{3 \times 2}$

3. Given that $F_{22} = 17711$ and $F_{24} = 46368$, find the following.

(a) F_{23} (b) F_{25} (c) F_{21} (d) F_{26} (e) F_{20}

4. Use the fact that $\phi^9 = 34\phi + 21$ to show that $\phi^{10} = 55\phi + 34$.

5. Suppose $\phi^{21} = a\phi + b$. Find a and b .

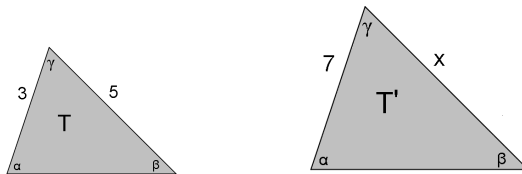
6. Use the quadratic formula to solve the following quadratic equations.

(a) $x^2 = x + 6$ (b) $2x^2 + 5x + 1 = 0$ (c) $6x^2 + 9x = -3$ (d) $5x^2 = 2x + 3$

7. Use the golden property to simplify the following expressions. Find the exact value of each without using a calculator.

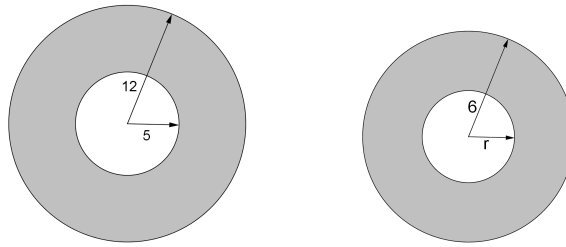
(a) $\sqrt{1 + \sqrt{1 + \sqrt{1 + \phi}}}$
 (b) $1 + \frac{1}{1 + \frac{1}{1 + (1/\phi)}}$

8. The triangles T and T' shown below are similar.

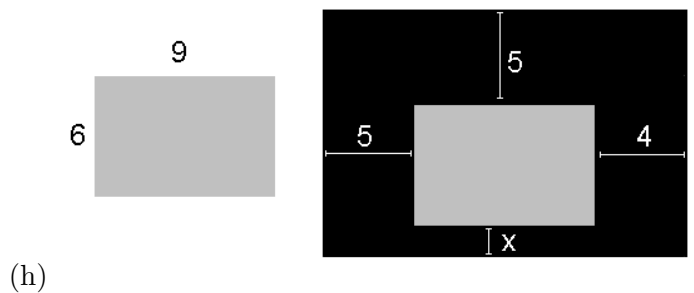
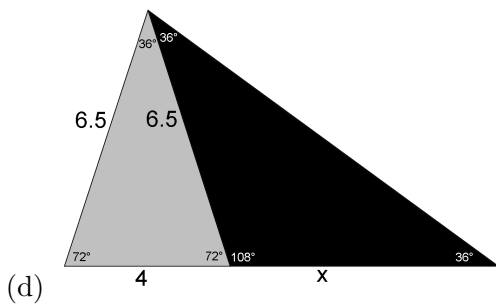
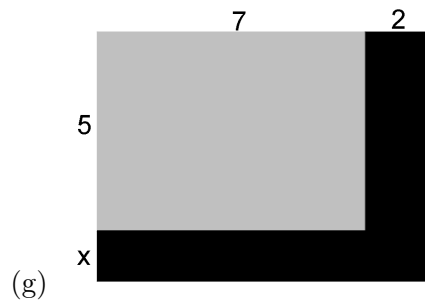
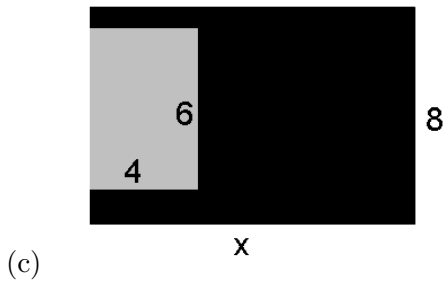
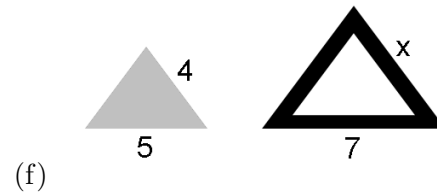
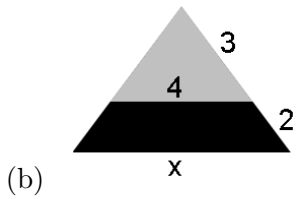
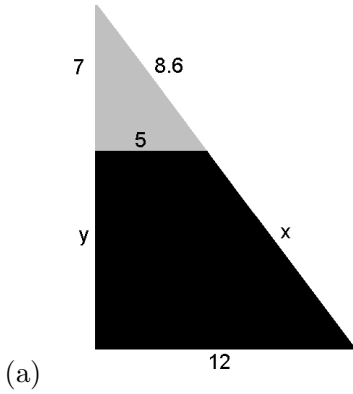


- (a) Find the length of the side labeled x in triangle T' .
- (b) If the perimeter of T is 12, find the perimeter of T' .
- (c) If the area of T is 7, find the area of T' .

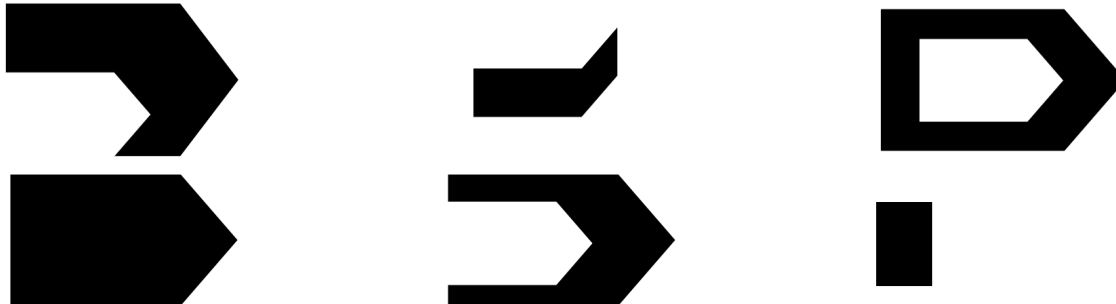
9. The circular rings R and R' shown below are similar. Find the value of r .



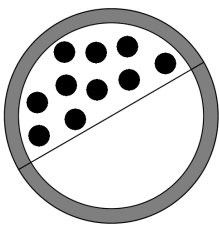
10. In the figures below solve for x (and y if applicable) so that the black shape is a gnomon to the grey shape. Note: the following shapes are drawn to scale.



11. Decide if each of the following figures are gnomons to the grey shape shown below.



12. Explain why it is difficult to tell golden rectangles and Fibonacci rectangles apart with the naked eye.
13. Clark and Lois jointly buy the half pepperoni-half cheese pizza shown for \$18. Suppose that Lois thinks that 40% of the value of the entire pizza is in the pepperoni half, and she thinks that 60% of the value is in the cheese half. Find the dollar value to Lois for each of the following pieces of pizza:



- (a) The cheese half of the pizza.
 - (b) The pepperoni half of the pizza.
 - (c) A slice of pizza which is $\frac{1}{5}$ of the cheese.
 - (d) A slice of pizza which is $\frac{1}{3}$ of the pepperoni.
 - (e) A slice of pizza which is $\frac{1}{2}$ of the pepperoni and $\frac{1}{4}$ of the cheese.
14. Four partners (Gibbs, Dinozzo, McGee and Todd) jointly own a piece of land with a market value of \$200,000. Suppose that the land is subdivided into four parcels (s_1, s_2, s_3, s_4). The partners are planning to split up, with each partner getting one of the four parcels. The following table gives the value of some of the parcels to each of the players.

	s_1	s_2	s_3	s_4
Gibbs	\$45,000		\$55,000	\$52,000
Dinozzo	\$49,000	\$50,000		\$49,000
McGee	\$53,000	\$51,000	\$47,000	
Todd		\$48,000	\$56,000	\$48,000

- (a) Fill in the missing values for each of the partners.
- (b) Which of the four parcels are fair shares to Gibbs?
- (c) Which of the four parcels are fair shares to Dinozzo?
- (d) Which of the four parcels are fair shares to McGee?
- (e) Which of the four parcels are fair shares to Todd?
- (f) Find a fair division of the land using s_1, s_2, s_3 and s_4 as fair shares.

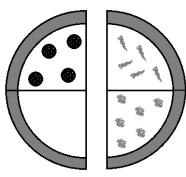
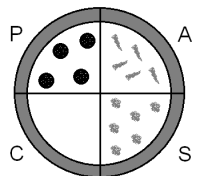
15. Four partners (Charles, Hank, Jean and Scott) jointly own a piece of land with a market value of \$400,000. Suppose that the land is subdivided into four parcels (s_1, s_2, s_3, s_4). The partners are planning to split up, with each partner getting one of the four parcels. Below is a partial table of the values each partner assigned to each parcel of land.

	s_1	s_2	s_3	s_4
Charles	\$115,000			\$135,000
Hank		\$100,000		
Jean			\$160,000	
Scott		\$92,000		

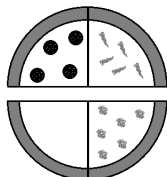
- (a) To Charles, s_2 and s_3 are equal in value. Fill in the missing values for Charles. Determine which of the four parcels are fair shares to Charles.
- (b) To Hank, s_1 is worth \$40,000 more than s_2 , s_4 is \$10,000 more than s_3 , and s_4 is \$15,000 less than s_2 . Fill in the missing values for Hank. Determine which of the four parcels are fair shares to Hank.
- (c) To Jean, s_1 is worth \$40,000 more than s_2 and \$20,000 more than s_4 , and s_3 is worth twice as much as s_4 . Fill in the missing values for Jean. Determine which of the four parcels are fair shares to Jean.
- (d) To Scott, s_2 and s_3 have equal value; and s_1, s_2, s_3 have a combined value equal to 70% of the value of the land. Determine which of the four parcels are fair shares to Scott.

16. Frank and Robert are planning to divide the pizza shown (right) using the divider-chooser method.

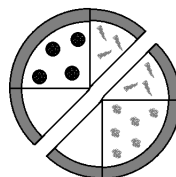
- Frank likes pepperoni, sausage and anchovies equally well, but hates cheese.
- Robert likes pepperoni, anchovies and cheese equally well, but hates sausage.



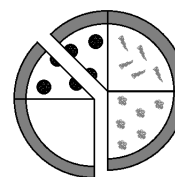
(i)



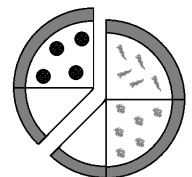
(ii)



(iii)



(iv)



(v)

- (a) For each of the cuts (i-v), decide if Frank would have made that cut as the divider. In the pizzas with cuts that Frank may have made, which of the two pieces is Robert's best choice?
- (b) For each of the cuts (i-v), decide if Robert would have made that cut as the divider. In the pizzas with cuts that Robert may have made, which of the two pieces is Frank's best choice?

17. Andy, Barney and Opie are attempting to fairly divide a half-apple half-peach pie (one of Aunt Bee's specialties!) using the lone-divider method. The following table gives the dollar value each one would place on each slice.

	s_1	s_2	s_3
Andy	\$1.75	\$1.75	\$1.75
Barney	\$1.50	\$2.00	\$2.50
Opie	\$1.00	\$1.50	\$1.25

- (a) Who was the divider?
 (b) What is each chooser's bid?
 (c) Give two possible fair-divisions.

18. Greg, Kim, Jimmy and Christine are attempting to fairly divide a half-Hawaiian half-barbecue chicken pizza using the lone-divider method. The following table gives some of the relative values of each slice to each player.

	s_1	s_2	s_3	s_4
Greg	40%		35%	15%
Kim		35%	25%	20%
Jimmy	25%	25%		
Christine	5%	5%	80%	

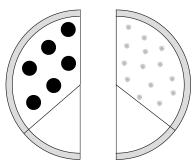
- (a) Fill in the missing values from the table.
 (b) Who was the divider?
 (c) What is each chooser's bid?
 (d) Give a possible fair-division.

19. In the three player lone-divider method, what happens if the two choosers both only want one of the pieces?

20. A, B, and C are dividing a pizza using the lone-chooser method. Their preferences are as follows.

- A loves pepperoni and sausage equally but hates cheese.
- B loves sausage and pepperoni equally but hates cheese.
- C loves sausage and cheese equally but hates pepperoni.

Suppose that A and C are the dividers and B is the chooser. In the first division, A cuts the pizza as shown below.



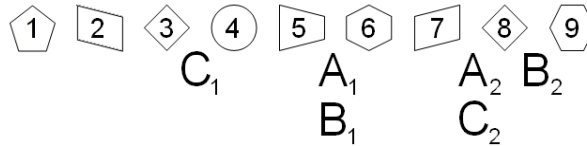
- (a) Describe which half C would pick.
 (b) Describe how C would subdivide his half.
 (c) Describe how A would subdivide her half.
 (d) Based on the subdivisions above, describe a possible final fair division of the pizza.

21. A, B and C are dividing three items using the method of sealed bids. Their bids on each of the items are given in the following table.

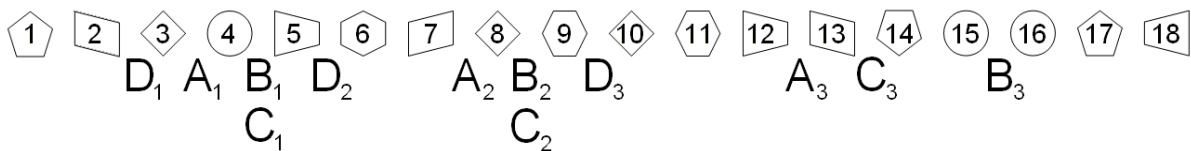
	A	B	C
Item 1	\$3,400	\$4,000	\$3,600
Item 2	\$500	\$580	\$600
Item 3	\$1,800	\$1,600	\$1,500

- (a) Find the value of each player's fair share.
 (b) Describe which players get what items (and how much money they pay or receive) in the first settlement.
 (c) Find the surplus after the first settlement is over.
 (d) Describe which players get what items (and how much money they pay or receive) in the final settlement.

22. After breaking off their engagement, Angelina and Brad agree to divide their common asset (a big screen HDTV) using the method of sealed bids. Angelina bids \$2100 for the TV, while Brad bids \$2200 for it. Describe the final settlement, that is, who gets the television and how much they must pay the other person.
23. Three players (A , B , and C) are dividing the array of 9 items shown below using the method of markers. The players' bids are indicated in the figure.



- (a) Which items go to A ?
- (b) Which items go to B ?
- (c) Which items go to C ?
- (d) Which items are left over?
24. Four players (A , B , C and D) are dividing the array of 18 items shown below using the method of markers. The players' bids are indicated in the figure.



- (a) Which items go to A ?
- (b) Which items go to B ?
- (c) Which items go to C ?
- (d) Which items go to D ?
- (e) Which items are left over?
25. Three players (A , B and C) are dividing 3 Snickers bars, 3 Nestle Crunch bars and 3 bags of Skittles using the method of markers.
- A only likes Crunch bars.
 - B likes Snickers bars and bags of Skittles equally, but hates Crunch bars.
 - C likes all three kinds of candy equally.



- (a) Place the markers for each player in the figure above.
- (b) Describe the allocation of candy to each player and which candy is left over.