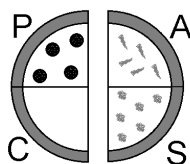


1. Monica, Joey, and Rachel are dividing a pizza using the lone-chooser method. Their preferences are as follows.

- Monica loves pepperoni and anchovies equally but hates cheese and sausage.
- Joey loves pepperoni and cheese equally but hates anchovies and sausage.
- Rachel loves pepperoni and sausage equally but hates cheese and anchovies.

Suppose that Monica and Joey are the dividers and Rachel is the chooser. In the first division, Monica cuts the pizza as shown below.



(a) Describe which half Joey would pick.

(b) Describe how Joey would subdivide his half into 3 pieces  $(j_1, j_2, j_3)$ .

$j_1$ : \_\_\_\_\_

$j_2$ : \_\_\_\_\_

$j_3$ : \_\_\_\_\_

(c) Describe how Monica would subdivide her half into 3 pieces  $(m_1, m_2, m_3)$ .

$m_1$ : \_\_\_\_\_

$m_2$ : \_\_\_\_\_

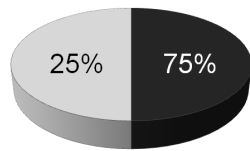
$m_3$ : \_\_\_\_\_

(d) Based on the subdivisions above:

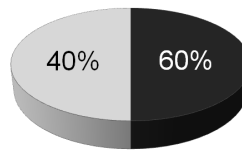
Which of Joey's pieces  $(j_1, j_2, j_3)$  would Rachel pick? \_\_\_\_\_

Which of Monica's pieces  $(m_1, m_2, m_3)$  would Rachel pick? \_\_\_\_\_

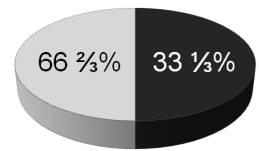
2. Arthur, Carrie and Doug are dividing the half-vanilla half-chocolate cake shown using the lone-chooser method. The figures below show the relative values of each half of the cake to each of them.



Arthur

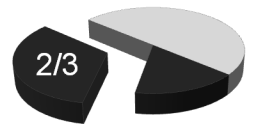


Carrie



Doug

Suppose that Arthur and Carrie are the dividers and Doug is the chooser. In the first division, Arthur cuts the cake as shown to the right ( $s_1$  is  $2/3$  of the chocolate;  $s_2$  is  $1/3$  of the chocolate and all of the vanilla).



- (a) Which of these pieces ( $s_1, s_2$ ) would Carrie choose? \_\_\_\_\_

- (b) Describe how Arthur could subdivide his piece into three equal shares ( $a_1, a_2, a_3$ ).

$a_1$ : \_\_\_\_\_

$a_2$ : \_\_\_\_\_

$a_3$ : \_\_\_\_\_

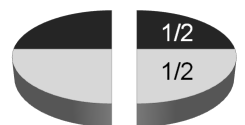
- (c) Describe how Carrie could subdivide her piece into three equal shares. ( $c_1, c_2, c_3$ )

$c_1$ : \_\_\_\_\_

$c_2$ : \_\_\_\_\_

$c_3$ : \_\_\_\_\_

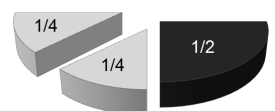
Suppose that Carrie and Doug are the dividers and Arthur is the chooser. In the first division, Doug cuts the cake vertically through the center as shown to the right.



- (d) Assuming Carrie split her half into three pieces as shown to the right ( $c_1$  is  $5/18$  of the chocolate,  $c_2$  is  $4/18$  of the chocolate and  $1/12$  of the vanilla,  $c_3$  is  $5/12$  of the vanilla), which piece would Arthur choose?



- (e) Assuming Doug split his half into three pieces as shown to the right ( $d_1$  is  $1/4$  of the vanilla,  $d_2$  is  $1/4$  of the vanilla,  $d_3$  is  $1/2$  of the chocolate), which piece would Arthur choose?



3. Serena, Blair and Chuck are dividing five items using the method of sealed bids. Their bids on each of the items are given in the following table.

	Serena	Blair	Chuck
Item 1	\$14,000	\$12,000	\$20,000
Item 2	\$24,000	\$15,000	\$33,000
Item 3	\$16,000	\$18,000	\$14,000
Item 4	\$16,000	\$15,000	\$18,000
Item 5	\$17,000	\$24,000	\$20,000

- (a) Find the value of each player's fair share.

Serena: \_\_\_\_\_

Blair: \_\_\_\_\_

Chuck: \_\_\_\_\_

- (b) Describe which players get what items (and how much money they pay or receive) in the first settlement.

Serena: \_\_\_\_\_

Blair: \_\_\_\_\_

Chuck: \_\_\_\_\_

- (c) Find the surplus after the first settlement is over.

- (d) Describe which players get what items (and how much money they pay or receive) in the final settlement.

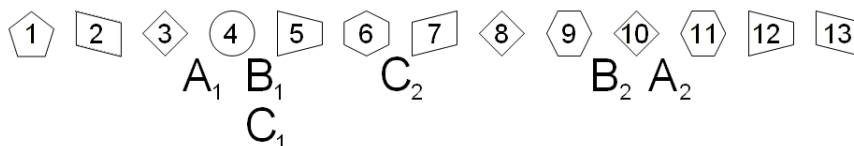
Serena: \_\_\_\_\_

Blair: \_\_\_\_\_

Chuck: \_\_\_\_\_

4. Sophia, Rose and Blanche jointly own a hotel. They can't get along anymore and decide to use the method of sealed bids, with the understanding that one of them will get the hotel and the other two will get cash. Sophia bids \$156,000, Rose bids \$150,000, and Blanche bids \$171,000. After the final settlement, how much money do Sophia and Rose get from Blanche for their one-third share of the hotel?

5. Three players ( $A$ ,  $B$ , and  $C$ ) are dividing the array of 13 items shown below using the method of markers. The players' bids are indicated in the figure.



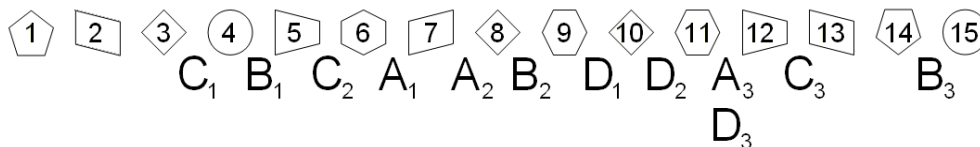
(a) Which items go to  $A$ ?

(c) Which items go to  $C$ ?

(b) Which items go to  $B$ ?

(d) Which items are left over?

6. Four players ( $A$ ,  $B$ ,  $C$  and  $D$ ) are dividing the array of 15 items shown below using the method of markers. The players' bids are indicated in the figure.



(a) Which items go to  $A$ ?

(d) Which items go to  $D$ ?

(b) Which items go to  $B$ ?

(e) Which items are left over?

(c) Which items go to  $C$ ?

7. Theo, Vanessa and Rudy are dividing 3 Snickers bars, 3 Nestle Crunch bars and 3 bags of Skittles. The following table shows the amount of money each player is willing to pay for each type of candy.

	Snickers	Crunch	Skittles
Theo	\$1.00	\$1.00	\$1.00
Vanessa	\$0.00	\$2.00	\$0.00
Rudy	\$1.50	\$0.75	\$0.75

Assume the candy is arranged as below:



- (a) Find the dollar value of each player's fair share.  
(Remember that there are 3 Snickers bars, 3 Crunch bars, and 3 bags of Skittles.)

Theo: \_\_\_\_\_

Vanessa: \_\_\_\_\_

Rudy: \_\_\_\_\_

- (b) Place the markers for each player in the figure above.  
(Use  $T_1, T_2$  for Theo,  $V_1, V_2$  for Vanessa, and  $R_1, R_2$  for Rudy.)
- (c) Describe the allocation of candy to each player and which candy is left over.

Theo: \_\_\_\_\_

Vanessa: \_\_\_\_\_

Rudy: \_\_\_\_\_

Surplus: \_\_\_\_\_

- (d) Suppose that the players decide to divide the leftover pieces by a random lottery in which each player gets to choose one piece. Suppose that Vanessa gets to choose first, Rudy second, and Theo last. Describe the division of the left over pieces.

Theo: \_\_\_\_\_

Vanessa: \_\_\_\_\_

Rudy: \_\_\_\_\_