Problem Set 2

(1) Reading: As I was a bit too ambitious in assigning reading last time, and we will have only two classes next week, you don't have to do any new reading. However, it may pay to re-read section 2.2 from Theorem 1 (page 23) to the end of Theorem 4 (page 27).

(2) Do problem 4 from section 2.5 of Evans.

(3) Suppose Ω is a smooth bounded open subset of $\mathbb{R}^n$, and suppose $u, v \in C^2(\Omega)$. Show that

$$\int_{\partial\Omega} v\partial_\nu u = \int_\Omega \nabla v \cdot \nabla u + \int_\Omega v \triangle u$$

and

$$\int_{\partial\Omega} (v\partial_\nu u - u\partial_\nu v) = \int_\Omega (v\triangle u - u\triangle v).$$

(4) Suppose $a_1, \ldots a_n$ are positive constants, and let $f \in C^2_c(\mathbb{R}^n)$. Find a solution to the equation $a_1 u_{x_1x_1} + \ldots + a_n u_{x_nx_n} = f$ on $\mathbb{R}^n$.

(5) Suppose a, b, c are constants such that all eigenvalues of the matrix

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

are positive. Let $f \in C^2_c(\mathbb{R}^2)$, and find a solution to the equation $au_{xx} + 2bu_{xy} + cu_{yy} = f$ on $\mathbb{R}^2$.

(6) Suppose Ω is an open subset of $\mathbb{R}^2$ such that $0 \notin \overline{\Omega}$, and define $\tilde{\Omega}$ to be the set

$$\tilde{\Omega} = \{x/|x|^2 \ \ |x \in \Omega\}.$$

Show that if u is harmonic on Ω, then the function $v(x) = u(x/|x|^2)$ is harmonic on $\tilde{\Omega}$.