Problem Set 5

(1) Reading: Read Section 2.3 from the beginning to the statement of Theorem 8 (page 59).

(2) Do problems 12, 13, 14, and 16 from section 2.5 of Evans.

(3) Let $g \in C(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$, and let $k > 0$ and $v \in \mathbb{R}^n$ be constant. Solve the problem

$$\begin{aligned} u_t - k\triangle u + v \cdot \nabla u &= 0 \text{ on } \mathbb{R}^n \times (0,\infty) \\ u(x,0) &= g(x). \end{aligned}$$

(4) Let $U \subset \mathbb{R}^n$ be a bounded domain with smooth boundary, and suppose $u_t - \triangle u = 0$ on $U \times (0,\infty)$ satisfies the boundary condition $\partial_\nu u = 0$ on $\partial U \times (0,\infty)$. Show that

$$\int_U u(x,t)dx$$

is constant in time. Why does this make sense physically?