1. Math 575 Final Review Sheet

The final exam covers all the material covered in class, with greater emphasis on the material covered between the start of infinite sequences and the end of infinite series – roughly the material covered between October 7 and December 7.

All major theorems and definitions we've covered in class are available in one of three sets of notes online. Twelve theorems (or groups of theorems) are starred: 3.6-3.7 and 5.8-5.9 in the notes on the reals; 2.2, 3.6-3.7 4.2, 5.10, and 5.11 in the notes on limits, derivatives, and integrals; and 1.2,1.12, 2.4, 3.7, and 4.2 in the notes on sequences and series. There is nothing particularly special about this list of theorems, but I think they provide a good overview of the course and an anchor for further study. Two of these (or parts of two of these) will make an appearance on the exam in some fashion. Nevertheless, **you should not limit your study to these theorems only**.

Some major themes to keep in mind while you study.

1.1. Topology of the Real Numbers.

Even though the exam has greater emphasis on the stuff we've done since then, much of the course is still about the real numbers and their topology, and there will still be exam questions about this.

In particular, be familiar with concepts like open and closed sets, limit points, supremum and infimum, connectedness, compactness, and continuity. Make sure you are familiar with the major theorems and their proofs.

Note also that some of these concepts come back in important ways later in the course (that's the point!) For instance, the Extreme Value Theorem plays an important role in the proof of the Mean Value Theorem, the Heine-Borel theorem plays an important role in the proof that a continuous function on $[a, b]$ is uniformly continuous, etc.

1.2. Sequences and Series.

Convergence is a major theme of this course, and it shows up both in the sense of limits and in the sense of sequences. Many of the proofs are stylistically similar, so it's a good exercise to compare them: for instance, look at the proof that $f(a_n)$ converges if f is continuous and a_n converges, and compare it to the proof that $f(g(x)) \to f(L)$ if f is continuous and $g(x) \to L$.

For sequences, be familiar with the definitions of convergence (both in terms of open sets and in terms of $\varepsilon - N$ definitions), accumulation points, subsequences, Cauchy sequences, and how to use them. For series, be familiar with the basic definitions, the elementary tests of convergence, and their proofs.

Much of the attention paid to sequences and series in this course is for the purpose of studying sequences and series of functions, and we will do more in this vein in Analysis 1 in the spring. For now, it's worth spending some time to wrap your head around uniform convergence and its basic consequences.

1.3. Limits, Derivatives, and Integrals.

One of the major goals of this course is for you to be familiar with the rigorous proofs of the main theorems of calculus. Make sure you know them! Try to be comfortable enough with the ideas and techniques that you can do similar problems.