1. Problem Set 11

Problem 1. *Suppose $a < c < b$. Show that f is integrable on $[a,b]$ if and only if it is integrable on $[a,c]$ and $[c,b]$, and that in that case*

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

Problem 2. *Suppose f and g are integrable on $[a,b]$. Show that $f+g$ is integrable on $[a,b]$, and*

$$\int_a^b (f+g)(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$$

Problem 3. *Suppose f is integrable on $[a,b]$, and F is defined by*

$$F(x) = \int_a^x f(x)dx.$$

Show that F is continuous.

Problem 4. *Suppose that f is integrable on $[a,b]$. Show there are continuous functions h and g such that $g \leq f \leq h$ on $[a,b]$ and*

$$\int_a^b h(x)dx - \int_a^b g(x)dx \leq \varepsilon.$$

Problem 5. *Suppose f' and g' are continuous. Show that*

$$\int_a^b f(x)g'(x)dx = f(x)g(x)|_a^b - \int_a^b f'(x)g(x)dx.$$

Problem 6. *Suppose f is twice differentiable on $[a,b]$ and $f(a) = f(b) = 0$, and moreover $f'' = f^3$ on $[a,b]$. Show that f must be the zero function.*

Problem 7. *Suppose f and g' are continuous. Show that*

$$\int_{g(a)}^{g(b)} f(u)du = \int_a^b f(g(x)) \cdot g'(x)dx.$$

Problem 8. *Define*

$$\log(x) = \int_1^x \frac{1}{t}dt.$$

Show that $\log(ab) = \log a + \log b$.

Problem 9. *Show that* $\lim_{x\to 0} \log x = -\infty$ *and* $\lim_{x\to\infty} \log x = \infty$.