1. The Last Problem Set

Problem 1. *Show that if* $\{a_n\}$ *is summable then* $\lim_{n\to\infty} a_n = 0$.

Problem 2. *Suppose* $0 \le a_n, b_n$ *for all* $n \in \mathbb{N}$*, and* $\lim_{n\to\infty} \frac{a_n}{b_n}$ *exists and is non zero. Show that* $\{a_n\}$ *is summable if and only if* $\{b_n\}$ *is summable.*

Problem 3. *Show that the sum* $\sum_{n=1}^{\infty} r^n$ *converges if* $0 \le r < 1$ *and diverges if* $r \ge 1$.

Problem 4. *Suppose* $0 \le a_n$ *for all* $n \in \mathbb{N}$*, and* $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = r$*. Show that* $\{a_n\}$ *is summable if* $r < 1$ *and not summable if* $r > 1$*. Show that* $r = 1$ *then* $\{a_n\}$ *may or may not be summable.*

Problem 5. *Suppose* f *is integrable on* $[a, b]$*. Show that*

$$\lim_{k\to\infty} \int_a^b f(x) \sin(kx)\, dx = 0.$$

Problem 6. *A sequence* $\{a_n\}$ *is called Cesaro summable, with Cesaro sum* L*, if*

$$\lim_{n\to\infty} \frac{s_1 + \ldots + s_n}{n} = L.$$

Show that if a sequence is summable then it is Cesaro summable.

Problem 7. *Find a sequence* $\{a_n\}$ *which is Cesaro summable, for which* $\lim_{n\to\infty} |a_n| = \infty$.

Problem 8. *Show that the sum*

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

converges for every x*. Define*

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

and show that e *is irrational.*