1. Problem Set 2

All references to "the notes" refer to the notes on the real numbers posted on the course webpage. If a question asks you to prove a proposition from the notes, you may freely use any *previous* proposition, lemma, theorem, corollary, etc. from the notes in your proof.

Otherwise you can use anything from the notes in your proof. In addition, you can use the results of previous problems in subsequent problems.

Problem 1. Prove Proposition 1.8 of the notes. Hint: for two open intervals (a, b) and (c, d) containing a common point x, show that the intersection of (a, b) and (c, d) is $(\max(a, c), \min(b, d))$.

Problem 2. Prove Theorem 1.10 of the notes.

Problem 3. Prove Proposition 2.2 of the notes.

Problem 4. Prove Proposition 2.3 of the notes.

Problem 5. Prove Corollary 2.4 of the notes.

Problem 6. Prove that any set of the form $\{x|x < a\}$ or $\{x|x > a\}$ is open.

Problem 7. Prove that a closed interval is closed.

Problem 8. Suppose $A \subset \mathbb{R}$ is nonempty. Show that p is a limit point of A if and only if, for any open set U containing p, $U \cap A \setminus \{p\} \neq \emptyset$.