1. Problem Set 3

All references to "the notes" refer to the notes on the real numbers posted on the course webpage. If a question asks you to prove a proposition from the notes, you may freely use any *previous* proposition, lemma, theorem, corollary, etc. from the notes in your proof.

Otherwise you can use anything from the notes in your proof. In addition, you can use the results of previous problems in subsequent problems.

Problem 1. *Prove Theorem 2.7 of the notes. (Notice that, in light of Corollary 2.10, this is really an if and only if statement!)*

Problem 2. *Prove Theorem 3.2 of the notes.*

Problem 3. *Prove Corollary 3.4 of the notes.*

Problem 4. *Prove that any open interval (a, b) is connected.*

Problem 5. *Suppose $z \in (a, b)$. Show that $(a, b) \setminus \{z\}$ is disconnected.*

Problem 6. *Suppose (a, b) is an open interval. Prove that $\sup(a, b) = b$ and $\inf(a, b) = a$.*

Problem 7. *Suppose $A, B \subset \mathbb{R}$, and for all $a \in A$ and $b \in B$, we have $a \leq b$. Prove that $\sup A \leq \inf B$.*

Problem 8. *Apply our definitions of open intervals, open sets, etc. to $\mathbb{Q}$. Prove that $\mathbb{Q}$ is disconnected.*