1. Problem Set 5

All references to "the notes" refer to the notes on the real numbers posted on the course webpage. If a question asks you to prove a proposition from the notes, you may freely use any *previous* proposition, lemma, theorem, corollary, etc. from the notes in your proof.

Otherwise you can use anything from the notes in your proof. In addition, you can use the results of previous problems (even on previous problem sets) in subsequent problems.

Problem 1. *Suppose $X \subset \mathbb{R}$ is connected. Show that the closure of X is connected.*

Problem 2. *Prove Corollary 4.8 of the notes.*

Problem 3. *Prove Proposition 5.4 of the notes.*

Problem 4. *Show that the union of finitely many compact sets is compact. Show that the intersection of arbitrarily many compact sets is compact. Give an example to show that the union of arbitrarily many compact sets may not be compact.*

Problem 5. *Suppose $X \subset \mathbb{R}$ is compact, and $x \notin X$. Show that $X \cup \{x\}$ is disconnected.*