1. Sample Prelim Problems

Problem 1. *Suppose* $g : (0,1) \to \mathbb{R}$ *is uniformly continuous. Show that* g *is bounded.*

Problem 2. *Suppose* $g : (0,1) \to \mathbb{R}$ *is uniformly continuous. Show that there exists a continuous* $f : [0,1] \to \mathbb{R}$ *such that* $g = f$ *on* $(0,1)$.

Problem 3. *Suppose*

$$f(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Q} \\ 1/q & \text{if } x = p/q \text{ with } p, q \in \mathbb{Z} \text{ in lowest terms.} \end{cases}$$

Show that f *is integrable on* $[0,1]$*, and calculate the integral.*

Problem 4. *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *is continuous and*

$$\int_a^b f(t)dt = 0$$

for all $a, b \in \mathbb{R}$. *Show that* $f(x) = 0$ *for all* x. *Give a counterexample if* f *is not continuous.*

Problem 5. *Consider the sequence defined recursively by* $a_1 = 1$ *and* $a_{n+1} = \sqrt{2a_n}$. *Determine whether or not this sequence converges, and if it does converge, determine what it converges to.*

Problem 6. *Suppose* $\{x_n\}$ *and* $\{z_n\}$ *are sequences which converge to* a*, and* f *is differentiable at* a. *Show that*

$$\lim_{n\to\infty} \frac{f(x_n) - f(z_n)}{x_n - z_n} = f'(a).$$

Problem 7. *Suppose* $\sum_{n=1}^{\infty} |a_n|$ *converges. Show that* $\sum_{n=1}^{\infty} |a_n|^2$ *converges.*