

MA 575 MIDTERM EXAM.

October 6 2017

Name: _____

Problem 1. *Suppose $X \subset \mathcal{C}$ has a supremum. Show that $\sup(X) = \sup(\overline{X})$.*

Problem 2. *Suppose $f : \mathcal{C} \rightarrow \mathcal{C}$ such that $f(X)$ is connected for each $X \subset \mathcal{C}$. Does it follow that f is continuous? Justify your answer.*

Problem 3. *Suppose X is compact and $f : \mathcal{C} \rightarrow \mathcal{C}$ is continuous. Show that $f(X)$ is compact.*

Problem 4. We define X to be dense in \mathcal{C} if every nonempty open set in \mathcal{C} contains some $x \in X$.

Suppose X is dense in \mathcal{C} and $f : \mathcal{C} \rightarrow \mathcal{C}$ is continuous, with $f(x) = c$ for all $x \in X$. Show that $f(x) = c$ for all $x \in \mathcal{C}$.

Problem 5. Suppose $\{[a_j, b_j]\}_{j \in \mathbb{N}}$ is a collection of non-empty closed intervals such that $[a_{j+1}, b_{j+1}] \subseteq [a_j, b_j]$. Show that there exists x such that $x \in \bigcap_{j \in \mathbb{N}} [a_j, b_j]$.

Extra Space