MA 575 Midterm Exam.

November 10 2017

Name: _____

Problem 1. *Suppose* $f(x) \leq g(x) \leq h(x)$ *for all* $x \in \mathbb{R}$ *and* $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$. *Show that* $\lim_{x \to a} g(x)$ *exists and is also equal to* L.

Problem 2. *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *has the property that* $|f(x)| \leq x^2$ *for all* $x \in \mathbb{R}$. *Show that* f *is differentiable at* 0.

Problem 3. *Suppose f is continuous on $[0, \infty)$ and*

$$\lim_{x \to \infty} f(x) = L$$

for some $L \in \mathbb{R}$. Show that f is bounded on $[0, \infty)$.

Problem 4. *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *is continuous everywhere and for all* $a, b \in \mathbb{R}$,

$$\int_a^b f(t)dt = 0.$$

Show that $f(t) = 0$ *for all* t.

Problem 5. *Suppose $f : \mathbb{R} \to \mathbb{R}$ is nondecreasing on $[0, 1]$; in other words if $x, y \in [0, 1]$ with $x \leq y$ then $f(x) \leq f(y)$. Show that f is integrable on $[0, 1]$.*

Extra Space