1. Problem Set 1

(You may use any result in the notes on the natural numbers, with the obvious exception that I expect you to prove the cancellation law by hand.)

Problem 1 (Cancellation Law)**.** *Suppose $n, m, p \in \mathbb{N}$, and $n+m = n+p$. Show that $m = p$.*

Definition 1 (Order)**.** *Let $n, m \in \mathbb{N}$. We define $n \leq m$ if $m = n+p$ for some $p \in \mathbb{N}$, and $n < m$ if $n \leq m$ and $n \neq m$.*

Problem 2. *Show that if $a, b, c \in \mathbb{N}$ and $a \leq b$ and $b \leq c$ then $a \leq c$.*

Problem 3. *Show that if $n, m \in \mathbb{N}$ then $n \leq m$ or $m \geq n$.*

Problem 4. *Suppose $n, m \in \mathbb{N}$ and $n < m$. Show that the statement $m < n$ is false.*

Definition 2. *Let $A \subset \mathbb{N}$. We say that a is a first point of A if $a \in A$, and $a \leq b$ for all $b \in A$. We say that a is a last point of A if $a \in A$ and $b \leq a$ for all $b \in A$.*

Problem 5. *Show that 0 is a first point of $\mathbb{N}$.*

Problem 6. *Show that every nonempty subset of $\mathbb{N}$ has a first point.*

Problem 7. *Show that not every nonempty subset of $\mathbb{N}$ has a last point.*

Definition 3 (Cardinality)**.** *Let $n \in \mathbb{N}$ and A be a set. We say A has n elements (or A has cardinality n) if there exists a bijective function*

$$f : A \to \{k \in \mathbb{N} | 1 \leq k \leq n\}.$$

We say A is finite if there exists $n \in \mathbb{N}$ such that A has cardinality n.

Definition 4 (Countability)**.** *Let A be a set. We say that A is countably infinite if there exists a bijective function*

$$f : A \to \mathbb{N}.$$

We say that A is countable if it is finite or countably infinite.

Problem 8. *Show that $\mathbb{N}$ is not finite.*

Problem 9. *Show that $A = \{(a, b) | a, b \in \mathbb{N}\}$ is countable.*