1. Problem Set 3

All references to "the notes" refer to the notes on the real numbers posted on the course webpage. If a question asks you to prove a proposition from the notes, you may freely use any *previous* proposition, lemma, theorem, corollary, etc. from the notes in your proof.

Otherwise you can use anything from the notes in your proof. In addition, you can use the results of previous problems in subsequent problems.

Problem 1. *Prove Theorem 3.9 of the notes. (Notice that, in light of Theorem 3.7, this is really an if and only if statement!)*

Problem 2. *Suppose* $x, y \in \mathcal{C}$, *and* $x < y$. *Show that there exists a* $z \in \mathcal{C}$ *such that* $x < z < y$.

Problem 3. *Suppose* $x \in \mathcal{C}$. *Show that* x *is a limit point of* $\mathcal{C}$.

Problem 4. *Prove that any closed interval* $[a, b]$ *is connected.*

Problem 5. *Suppose* $A \subset \mathcal{C}$ *is connected. Show that* $\overline{A}$ *is connected.*

Problem 6. *Give an example of a set* A *such that* A *is not connected, but* $\overline{A}$ *is. (Justify your example!)*

Problem 7. *Suppose* (a, b) *is an open interval. Prove that* $\sup(a, b) = b$ *and* $\inf(a, b) = a$.

Problem 8. *Suppose* $A, B \subset \mathbb{R}$, *and for all* $a \in A$ *and* $b \in B$, *we have* $a \leq b$. *Prove that* $\sup A \leq \inf B$.