1. Problem Set 5

All references to "the notes" refer to the notes on the continuum posted on the course webpage. For this problem set you are freely allowed to use any result in the notes *before* (not including) Theorem 6.8, the Heine-Borel Theorem.

As always you can use the results of previous problems in subsequent problems.

Problem 1. *Suppose $X \subset \mathcal{C}$ is compact and $Y \subseteq X$ is closed. Show that Y is compact.*

Problem 2. *Show that the arbitrary intersection of compact sets is compact.*

Problem 3. *Show that the finite union of compact sets is compact. Give a counterexample for arbitrary unions.*

Problem 4. *Define* closed cover *analogously to our definition of open cover, and define X to be supercompact if every closed cover of X contains a finite subcover. Show that X is supercompact if and only if it is finite.*

Problem 5. *Suppose $X \subset \mathcal{C}$. Define X to be intercompact if X has the following property: for any collection of closed sets $\{E_\lambda\}_{\lambda \in L}$ with the property that any finite intersection $E_{\lambda_1} \cap \ldots \cap E_{\lambda_n} \cap X \neq \varnothing$, we have $\cap_{\lambda \in L} E_\lambda \cap X \neq \varnothing$.*

Show that X is intercompact if and only if it is compact.