1. Problem Set 8

Problem 1. *Suppose* $a < b < c$. *Show that* f *is integrable on* $[a, c]$ *if and only if it is integrable on* $[a, b]$ *and* $[b, c]$, *and that*

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx.$$

Problem 2. *Suppose* f *and* g *are integrable on* $[a, b]$. *Show that* $f + g$ *is integrable on* $[a, b]$ *and*

$$\int_a^b (f+g)(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$$

Problem 3. *Suppose* g *is integrable on* $[a, b]$. *Show that for any* $\varepsilon > 0$ *there are continuous functions* f *and* h *on* $[a, b]$ *such that* $f \leq g \leq h$ *on* $[a, b]$ *and*

$$\int_a^b h(x)dx - \int_a^b f(x)dx \leq \varepsilon$$

Problem 4. *Suppose that* f *and* g *are differentiable on* $[a, b]$ *and* f' *and* g' *are continuous there. Show that*

$$\int_a^b f(x)g'(x)dx = f(x)g(x)|_{x=a}^b - \int_a^b f'(x)g(x)dx.$$

Problem 5. *Suppose that* f *and* g' *are continuous. Show that*

$$\int_{g(a)}^{g(b)} f(u)du = \int_a^b f(g(x)) \cdot g'(x)dx.$$

Problem 6. *For* $x > 0$ *define*

$$\log(x) = \int_1^x \frac{1}{t}dt.$$

Show that $\log(ab) = \log(a) + \log(b)$.