MA 575 Sample Final Exam.

December 11 2017

Name: _____

Problem 1. *Suppose $A \subset \mathbb{R}$ is connected and $f : \mathbb{R} \to \mathbb{R}$ is continuous. Show that $f(A)$ is connected.*

Problem 2. *Let*

$$f(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Q} \\ 1/q & \text{if } x = p/q \text{ in lowest terms.} \end{cases}$$

Show that f is integrable and $\int_0^1 f(x)dx = 0$.

Problem 3. *Suppose f is uniformly continuous on $(0, 1)$. Show that the sequence*
$$f(1), f(1/2), f(1/3), f(1/4), \ldots$$
converges. Is this still true if "uniformly continuous" is replaced by "continuous"?

Problem 4. *Suppose $\{a_n\}$ is a positive sequence with*

$$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = r$$

Show that if $r > 1$ then $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.

Problem 5. *Suppose $\{a_n\}$ is a nonnegative sequence with no accumulation points. Show that*

$$\lim_{n\to\infty} a_n = \infty.$$

Extra Space