1. Worksheet Sep 20

Definition 1.1. *Let $X \subseteq \mathcal{C}$, and suppose $\mathcal{O} = \{U_\lambda\}$ is a collection of subsets of $\mathcal{C}$. We say $\mathcal{O}$ is an open cover of $\mathbb{R}$ if i) all the U_λ are open and ii)*

$$X \subset \bigcup_\lambda U_\lambda.$$

Definition 1.2. *Let X be a subset of $\mathcal{C}$. X is* compact *if for every open cover $\mathcal{O}$ of X, there exists a finite subset $\mathcal{O}' \subset \mathcal{O}$ that is also an open cover.*

Exercise 1.1. *If X is compact and $A \subset X$, does it follow that A is compact?*

Exercise 1.2. *Are unions of finitely many compact sets compact? What about unions of arbitrarily many compact sets? Justify your answers.*

Exercise 1.3. *Are intersections of finitely many compact sets compact? What about intersections of arbitrarily many compact sets? Justify your answers.*

Recall that last time we proved the following lemma:

Lemma 1.4. *Let $p \in \mathcal{C}$ and consider the collection:*

$$\mathcal{O} = \{\text{ext}\,(a,b) \mid p \in (a,b)\}.$$

Then $\mathcal{O}$ covers $\mathcal{C} \setminus \{p\}$ but no finite subcollection of $\mathcal{O}$ covers $\mathcal{C} \setminus \{p\}$.

Exercise 1.5. *Use the lemma to prove that if X is compact then it is closed.*

Exercise 1.6. *Prove that the closed interval $[a,b]$ is compact.*