Problem Set 8

(1) Reading: Section 6.3 from Theorem 4 (p334) up until the end of the section, and Section 6.4 up to the end of Theorem 2 (page 347).

(2) Do question 8 and 12 from Section 6.6 of Evans.

(3) Suppose Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary, and let $\Gamma \subset \partial\Omega$ be an open subset of $\partial\Omega$. Suppose $\triangle u = 0$ on Ω, and $u = \partial_\nu u = 0$ on Γ. Show that $u \equiv 0$ on Ω. Hint: Let Ω_2 be a bounded domain with smooth boundary in $\mathbb{R}^n$ which contains Ω, such that $\partial\Omega \setminus \Gamma \subset \partial\Omega_2$. Extend u to be a function on Ω_2 with $u \equiv 0$ on $\Omega_2 \setminus \Omega$. What's special about this extension?

(4) Suppose U is a smooth bounded domain in $\mathbb{R}^n$, and $u \in C(\overline{\Omega})$ has the property that

$$\int_\Omega u \triangle v = 0$$

for all $v \in C_c^\infty(\Omega)$. Does it follow that u is harmonic on Ω? Hint: This is hard.