1. Problem Set 1

Problem 1. *Show that* $m^*((0,1)) = 1$.

Problem 2. *Suppose* $A \subset \mathbb{R}$ *and* $h \in \mathbb{R}$. *Define* $A + h = \{a + h | a \in A\}$. *Show that* $m^*(A) = m^*(A + h)$.

The next two questions consider the Cantor set E defined by the following process: Define $E_0 = [0, 1]$, and $E_1 = [0, 1/3] \cup [2/3, 1]$, so E_1 consists of E_0 with the middle third removed. Continue the process, defining E_{j+1} by removing the middle third of each interval in E_j: so $E_2 = [0, 1/9] \cup [2/9, 1/3] \cup [2/3, 7/9] \cup [8/9, 1]$, etc. Finally, define

$$E = \cap_{j=1}^{\infty} E_j.$$

Problem 3. *Show that* $x \in E$ *if and only if* x *is a limit point of* E. *Note in particular that* E *is closed.*

Problem 4. *Show that* $m^*(E) = 0$.

Problem 5. *Consider the set* F *defined in the same manner as* E, *except that at the* k^{th} *step we remove not the middle third but the middle interval of length* 2^{-2k}. *(This is sometimes called a* fat Cantor set*.) Show that* $m^*(F) = 1/2$.

Problem 6. *Suppose* $m^*(A) = 0$. *Show that* A *is measureable.*

Problem 7. *Suppose that* $B \subset \mathbb{R}$ *has the property that for every* $\varepsilon > 0$ *there exist measureable* $A, C \subset \mathbb{R}$ *such that* $A \subset B \subset C$ *and* $m(C \cap A^c) < \varepsilon$. *Show that* B *is measureable.*