COMPUTATION WITH POWER SERIES AND FORMAL GROUP LAWS

NATHANIEL STAPLETON

1. Plan

Power series arise naturally in essentially every area of mathematics. However, working with power series using a computer is challenging. By definition each power series is equipped with an infinite amount of data: its coefficients. The purpose of this project is to get a computer to lazily calculate with power series and formal group laws. This means that we should be able to be able to specify and manipulate power series without calculating any coefficients. Calculation should only occur when the user finally asks to see the coefficients of a power series and then the program should only calculate precisely what is needed to see those coefficients. Key partial goals include manipulating power series defined over a fixed commutative ring, computing multiplicative inverses of unit power series, calculating the universal formal group law and the Honda formal group laws, and extracting the $[i]$-series from a formal group law. Ideally we should be able to independently verify all of Pearson's computer calculations in "Calculating formal group laws" (not available online, ask me for a copy).

Stages in this project include:

- Choose a programming language and/or algebra software with which to work. Mid term goals can be realized over the integers and $\mathbb{F}_p$. Longer term goals require the ability to work with more complicated commutative rings. Consider using Haskell.
- Understand how to specify a power series (in one or many variables). Create a power series data type.
- Program operations such as composition, multiplication, addition, derivatives, and multiplicative and composition inverses of power series.
- Understand how to specify and manipulate formal group laws. Program operations such as the $[i]$-series, conjugation of formal group laws, and the relative Frobenius.
- Specify the universal formal group law, the universal p-typical formal group law, the Honda formal group law, and the universal deformation formal group law.

Math to learn along the way:

- Power series and Weierstrass's preparation theorem.
- Formal group laws and Lazard's theorem.
- Monads in algebra and computer science.