LAB PROJECT:
COMPUTATIONS IN TROPICAL GEOMETRY

Tropical geometry is a relatively new offshoot of algebraic geometry and combinatorics where the usual real number operations are replaced with their "tropicalizations": $x \oplus y = MIN\{x, y\}$, $x \otimes y = x + y$. It turns out that these operations work well enough with each other that we can talk about solving tropical polynomial equations. The set of solutions to a tropical equation is called a tropical variety, and the structure of tropical varieties is the subject of tropical geometry.

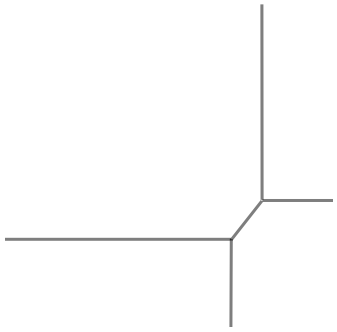

FIGURE 1. This nifty picture looks like it's probably the tropical solutions to $2 \oplus x \oplus y \oplus 2 \otimes x \otimes y$.

It's even possible to say things about the solutions to a normal polynomial by studying its tropicalization, namely the tropical polynomial obtained by subsituting normal operations for tropical operations. The tropicalization can be thought of as a combinatorial avatar of the orginal polynomial, and in a certain sense it contains information about the solutions to the polynomial "at infinity."

The goal of this project is to compute tropical varieties, study them, and draw pictures of them. This will involve getting familiar with a few different computer packages (e.g. Macaulay2, Polymake, 4ti2, and Mathematica), as well as the basics of algebraic geometry and computer algebra. We'll be looking for tropical varieties with "prime cones" and varieties which are "well-poised"; I will tell you more about these properties as the project progresses.